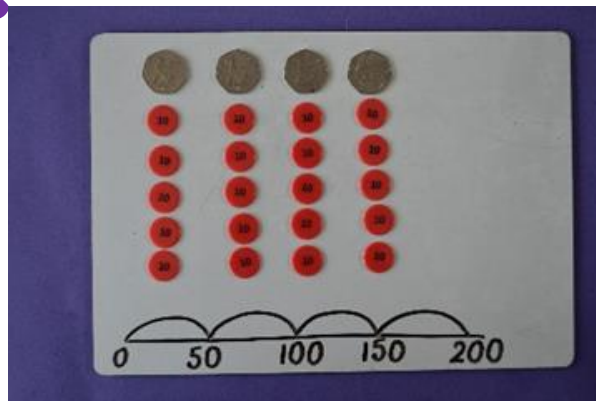
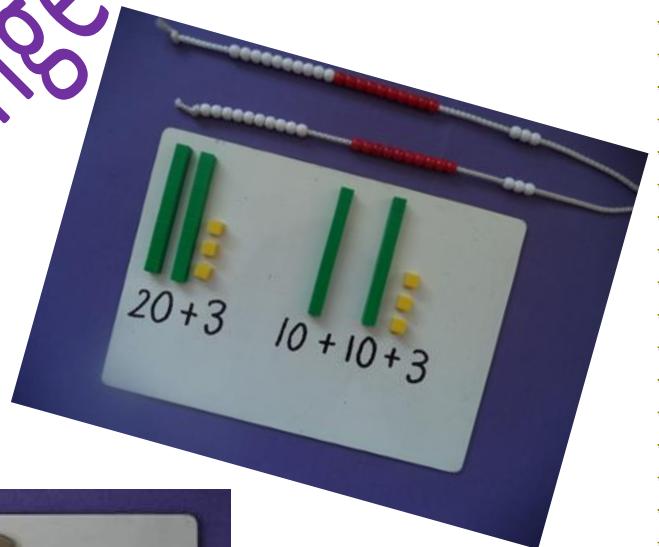


Ash Grange Nursery & Primary School.

Copyright of Ash Grange Primary School



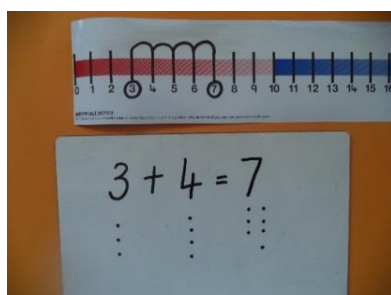
Parent Workshop
Calculation

Addition

There are **two** main structures:



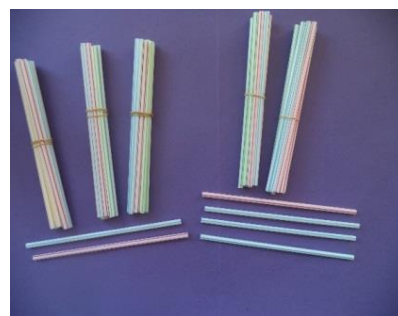
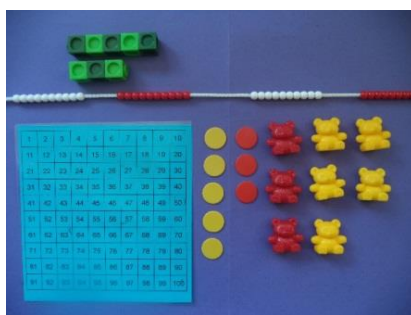
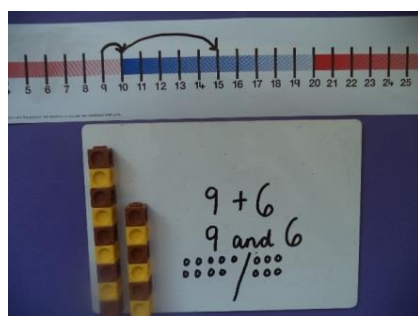
The Collecting Together Structure: union of two sets



Counting on and increasing structure

Progression in Addition Year 1

1. Children experience combining two then *more than two* groups of objects using counting on and the language of addition: **plus**, **add**
2. Children experience increasing numbers: **What is two more than seven?**
3. They compare quantities to say how many less/how many more.



Use practical resources such as toys, counters, cubes, straws and numberlines/hundred grids. This will encourage children to count in groups rather than ones.

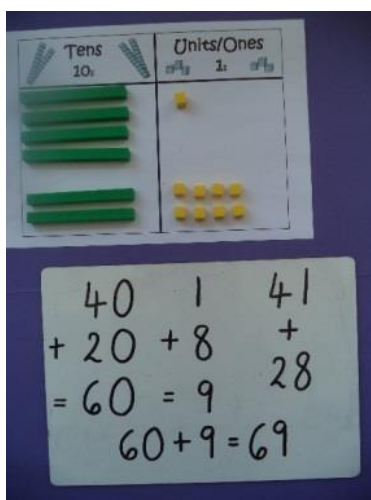
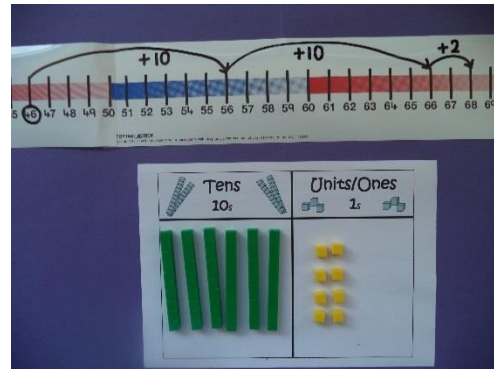
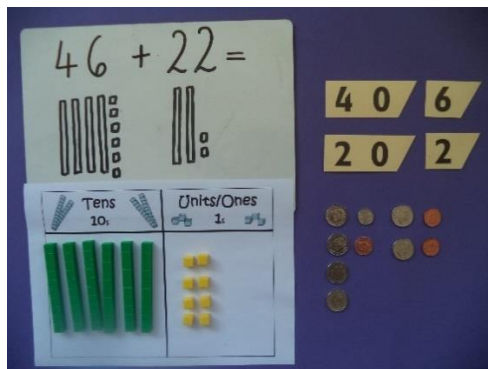
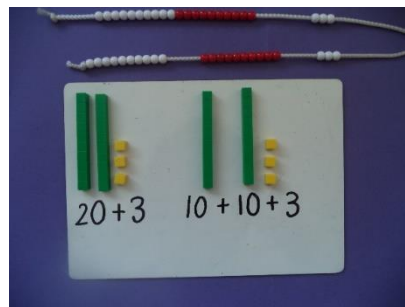
Fluency-mental maths targets for addition

- Count forwards and backwards, to and across 100 beginning with 1 or 0 from any number.
- Switch count between tens and ones e.g 10, 20, 30, 31, 32, 33
- Represent and use number bonds to 20 and the relationship between addition and subtraction
- Find one more than a given number
- Find ten more than a given number
- Count in multiples of 2s, 5s and 10s, starting on multiples to highlight pattern recognition



Year 2

Children should be able to partition numbers in different ways e.g $2+2+2+1$ or $5+3=$ or 23 as $20+3$ or $10+13$

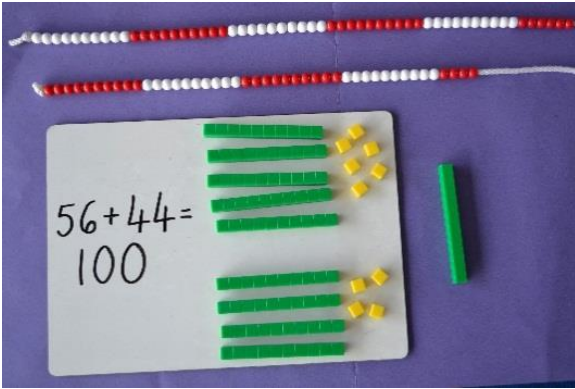


Children must have opportunities to add more than two numbers e.g $4+23+40$

Children should understand the language of 'sum'

Ensure that children understand that addition is commutative (can be added in any order) $4+5=9$ $5+4=9$

Fluency-mental maths targets for addition



- Show increasing fluency in deriving pairs of numbers up to 10 and then up to 20
- Use knowledge to derive and use number facts up to 100 (see photo)
- Add numbers mentally including TU+U, TU+tens, TU+TU, U+U+U

Year 3

Children add numbers with up to 3 digits (leading to formal written column method)

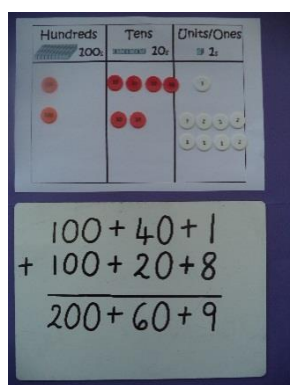
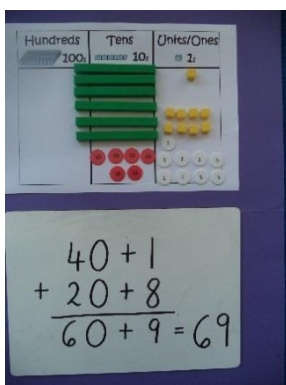
Children must have opportunities to add more than two numbers with different number of digits e.g HTU+TU+TU 450+45+43

Children should partition numbers to 1000 in different ways

They solve problems in different contexts including missing number problems



Expanded recording without exchange



The number is partitioned into hundreds, tens and ones using deines or place value counters and placed on the place value mat. The sum is written alongside the apparatus in the expanded form.

Hundreds	Tens	Units/Ones
100s	10s	1s
	●●●●	●●●
	●●	●●●●
		●●●●

$$\begin{array}{r}
 40 + 3 \\
 20 + 8 \\
 \hline
 70 + 1 \\
 10
 \end{array}$$

Expanded recording with exchange

Step 1-Place value counters/deines are counted out and placed on the place value mat in the correct column.

Hundreds	Tens	Units/Ones
100s	10s	1s
	●●●●	●
	●●	
	●	

$$\begin{array}{r}
 40 + 3 \\
 20 + 8 \\
 \hline
 70 + 1 \\
 10
 \end{array}$$

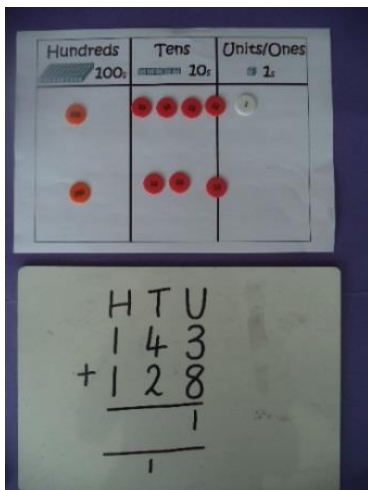
Step 2-Ten ones are exchanged for a ten counter/ten stick and notation written alongside. The exchanged ten is written in full beneath the tens. This will help the children to see the ten as 10 and not as '1'. The tens are then added together including the exchanged ten.

Hundreds	Tens	Units/Ones
100s	10s	1s
●	●●●●	●
●	●●	●●●●
		●●●●

$$\begin{array}{r}
 \text{HTU} \\
 141 \\
 + 128 \\
 \hline
 269
 \end{array}$$

Compact column recording (no exchanging)

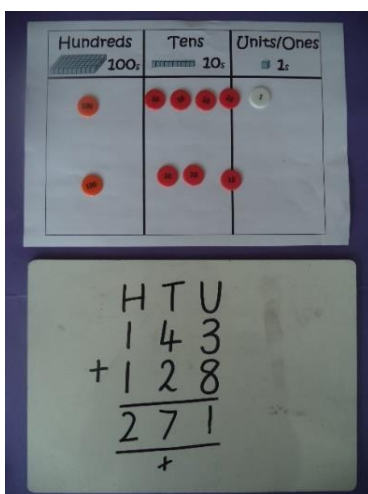
Place value counters are placed onto the mat and added beginning with the ones first.



Compact column recording (with exchanging)

Step 1 - Place value counters are counted out and placed on the place value mat and the addition is written alongside. The ones, tens and hundreds must be lined up in the correct columns. HTU can be written above the columns in the early stages until the children have understood the significance of the place value.

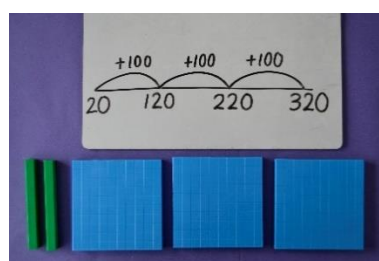
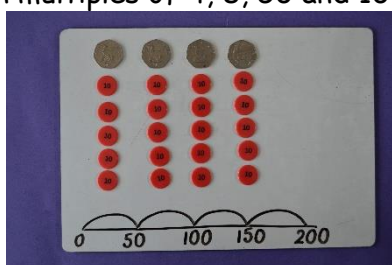
Step 2 - Ten ones are exchanged for a ten counter/ten stick and this is recorded as a line underneath the tens column.



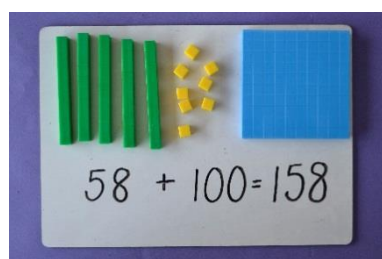
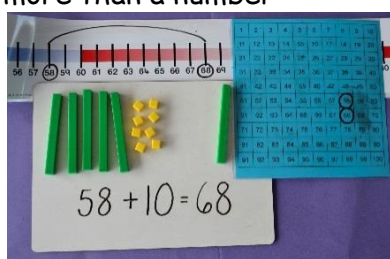
Step 3 - The tens are added together including the exchanged ten. This is then crossed out so that it is not added twice by mistake. The hundreds are added to complete the sum.

Fluency-mental maths targets for addition

- Count in ones, tens and hundreds maintaining fluency
- Count from 0 in multiples of 4, 8, 50 and 100



- Find 10 or 100 more than a number



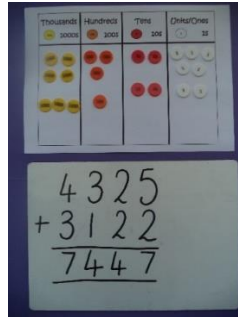
- Mentally add HTU+ones, HTU+tens, HTU+hundreds
- Perform mental calculations with two-digit numbers where the answer is more than 100

Year 4

1. Add numbers with up to 4 digits (formal written column method) including numbers with up to two decimal places in the context of money
2. Children must add more than two numbers including decimals with a differing number of digits
3. Solve two-step problems in different contexts including missing number problems

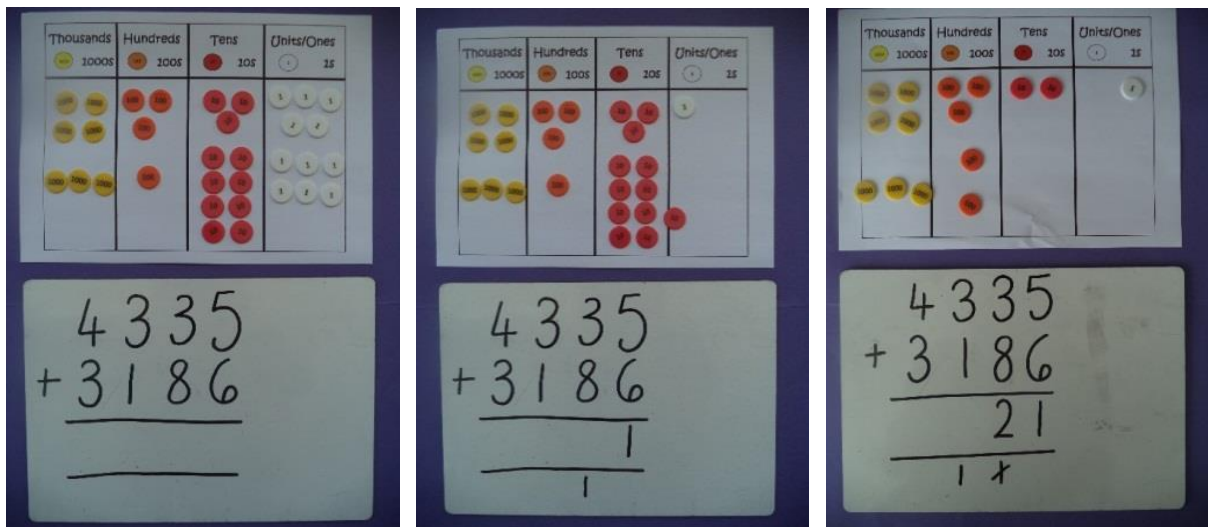
Apparatus should be used alongside recording

Four-digit column addition (no exchange)



Place value counters are used alongside the written calculation. The digits are lined up in the columns correctly.

Four-digit column addition (exchanging)



The place value counters are counted out onto the place value mat. The counters are exchanged and the calculation written alongside. As the exchanged digits are added, they are crossed off.

Thousands	Hundreds	Tens	Units/Ones
1000s	100s	10s	1s
4	3	3	5
3	1	8	6
7	5	2	1

$$\begin{array}{r}
 \text{Th} \\
 4335 \\
 + 3186 \\
 \hline
 7521 \\
 \hline
 \end{array}$$

The thousand digit is marked with a 'Th' as the children use this digit in their addition for the first time.

Tens	Units/Ones	Tenths	Hundredths
10s	1s	1/10 or 0.1	1/100 or 0.01
2	5	1	
3	2	3	
5	7	4	

$$\begin{array}{r}
 2.51 \\
 + 3.23 \\
 \hline
 5.74
 \end{array}$$

Decimal addition up to 2 places (no exchanging)

Use the decimal place value mat and place value counters (tenths and hundredths) to use alongside the written algorithm.

Adding decimals (with exchanging)

Tens	Units/Ones	Tenths	Hundredths
10s	1s	1/10 or 0.1	1/100 or 0.01
	7	8	9
	6	4	2

$$\begin{array}{r}
 7.89 \\
 + 6.42 \\
 \hline
 \hline
 \hline
 \end{array}$$

Tens	Units/Ones	Tenths	Hundredths
10s	1s	1/10 or 0.1	1/100 or 0.01
	7	8	9
	6	4	2
			1

$$\begin{array}{r}
 7.89 \\
 + 6.42 \\
 \hline
 1 \\
 \hline
 1
 \end{array}$$

Tens	Units/Ones	Tenths	Hundredths
10s	1s	1/10 or 0.1	1/100 or 0.01
	7	8	9
	6	4	2
		3	1

$$\begin{array}{r}
 7.89 \\
 + 6.42 \\
 \hline
 31 \\
 \hline
 1
 \end{array}$$

Tens	Ones/Ones	Tenths	Hundredths
● 10s	○ 1s	● 1/10 or 0.1	● 1/100 or 0.01
●	○ ○ ○ ○	● ● ● ● ● ●	●

$$\begin{array}{r}
 7.89 \\
 + 6.42 \\
 \hline
 14.31 \\
 \hline
 \end{array}$$

Exchanging of tenths, ones and tens alongside the written algorithm as with whole numbers

Fluency-mental maths targets for addition

- Count in 6s, 7s, 8s, 9s, 25s and 100s
- Find 1000 more than a number

241 + 1000 = 1241

241 1241 2241 3241

- Perform mental calculations with increasingly large numbers to aid fluency

Year 5

1. Add numbers with more than four-digits and decimals to three places (formal written column method)
2. Solve multi-step problems selecting and justifying methods
3. Perform mental calculations with increasingly large numbers

Column addition (no exchanging)

$$\begin{array}{r} 21.41 \\ + 1.12 \\ + 0.35 \\ \hline 22.88 \end{array}$$

$$\begin{array}{r} 2141 \\ + 1128 \\ \hline 3269 \end{array}$$

Column addition (with exchanging)

$$\begin{array}{r} 5189 \\ + 3128 \\ \hline 8317 \\ \hline + + \end{array}$$

$$\begin{array}{r} 51.89 \\ + 3.128 \\ \hline 55.018 \\ \hline + + \end{array}$$

The children should be asked to solve additions where the place value is not always the same. For example, the bottom number has three decimal places and the top number has two.

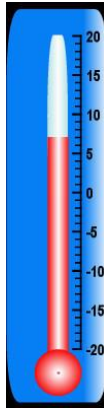
Fluency-mental maths targets for addition

- Count forwards in powers of ten up to 100000



Daily counting with the place value chart

- Count forwards in positive and negative numbers through zero-*The representation of the thermometer is useful for this (see Collins online resources)*



- Practice mental calculations with increasingly large numbers

Year 6

Add numbers with more than four-digits and decimals up to three places (formal written column method)

Solve more complex calculations mentally

Solve multi-step problems in contexts, deciding which operations and methods to use and why

Fluency-mental maths targets for addition

- Count in tens and hundreds increasing the fluency of order and place value
- Perform increasingly complex mental calculations and those with increasingly large numbers.

Subtraction

What is involved in understanding subtraction?

There are **five** possible structures for subtraction.

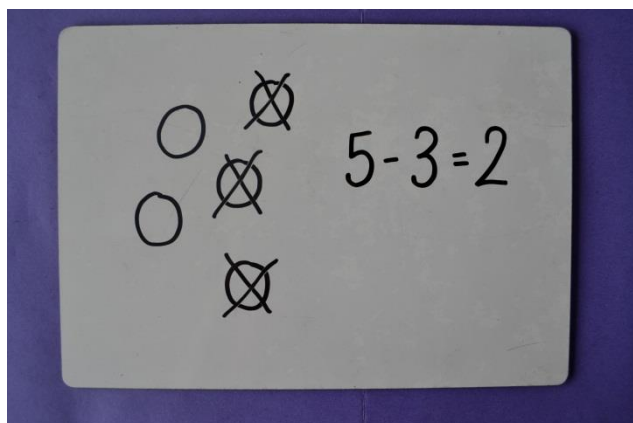
The partitioning structure-take away

This structure is the most commonly referred to.

'There are 12 birds sitting on a fence. 3 fly away. How many are left?'

'I had 12 chocolates and I ate 3 of them. How many are there now?'

The partitioning structure involves there being a number of objects of which some are then partitioned off or removed.



This structure can be used in measuring contexts too. For example,

'I had 12p and I spent 3p'

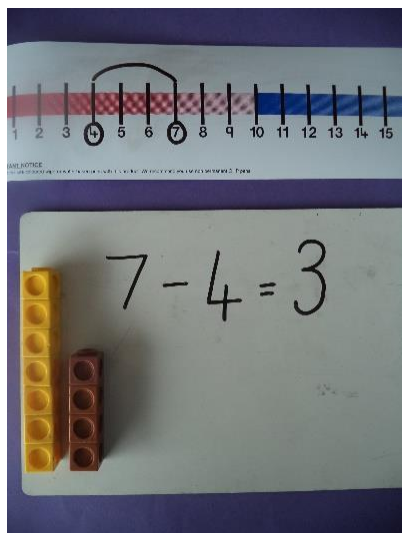
'There were 12 litres of wine in my cellar. We drank 3 litres'

Partitioning structure is **not the only** structure and subtraction should **not be a synonym** for 'take away'. This structure is a very limited one.

The comparison structure

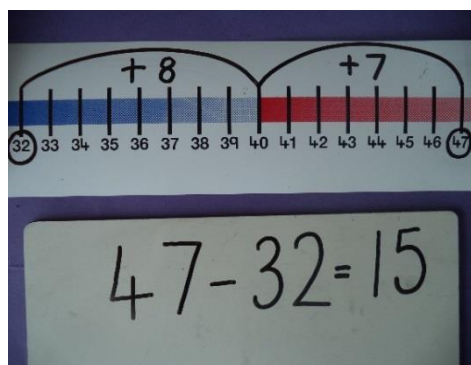
'If my friend is 12 and his brother is 3, how much older is he?'

A very important structure of subtraction, quantities that are compared.



These are the 'finding the difference' questions that children find difficult to unpick. They will often add the numbers together and do not naturally associate subtraction or - sign with finding the difference. This is because there is an over-emphasis on - meaning 'take away'.

But this structure is useful because it means that children can count on to solve a subtraction e.g. $47 - 32 = 15$



Children can also use their number bonds to subtract rather than go through the complicated process of counting back.

Vocabulary associated with comparison/finding the difference

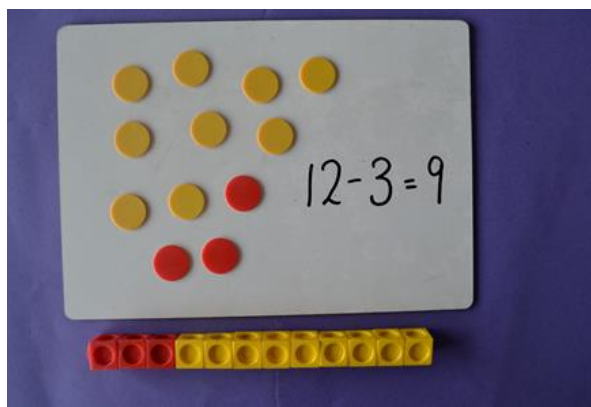
Greater	Less
More	Fewer
Larger	Smaller
Longer	Shorter
Higher	Lower
Further	Nearer
Fatter	Thinner
Heavier	Lighter
Holds more	Holds less
Costs more	Costs less
Later	Earlier
Older	Younger

The words in the **left** hand column tend to be used more so it is important to give children experience of both columns. It comes more naturally to a child to think they have 'more' sweets than their friend, rather than their friend having 'fewer' sweets than them.

The complement of a set structure

'There were 12 soldiers. 3 were ill. How many were **not** ill?'

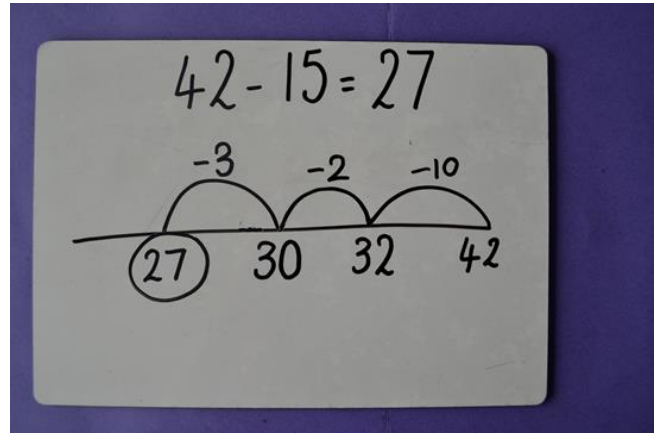
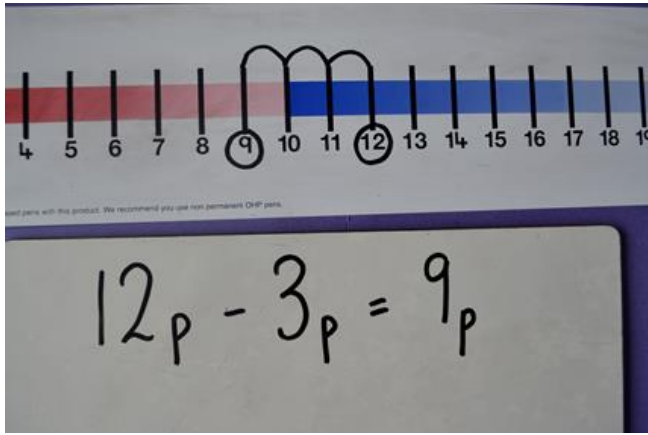
Making cube towers that are two or more different colours or using the reversible counters can help children to visualise this type of question.



The reduction structure: counting back

'The price of a chocolate bar is 12p. The shopkeeper took 3p off the price'.

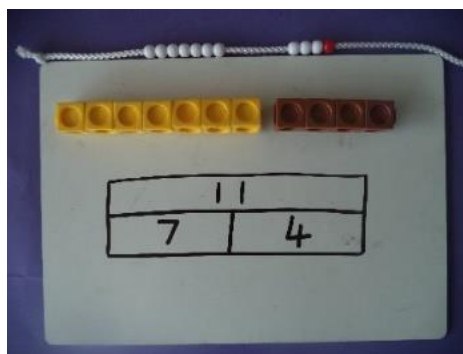
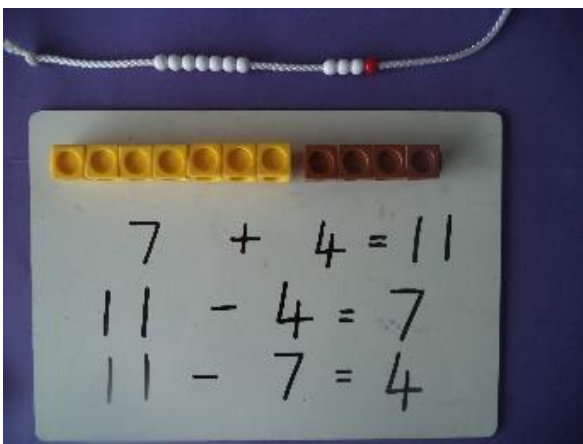
The price of the chocolate bar had reduced by 3p. Complications arise when the children count the starting number as one of the jumps back. To overcome this use number-line to support counting back with the language '12, then 3 less' so that they keep the starting number in their head, then count back the correct number.



Begin by using number tracks, then marked number-lines or hundred squares before progression to unmarked. The children can then partition the number they want to subtract and jump in groups of tens and ones.

The inverse of addition structure

In many ways, this is the **most important** and fundamental subtraction structure.



I have 7p and I need 11p. How much more money do I need?

Children may add these numbers together. Using concrete structures such as cube towers,

number-lines, fact family triangles, Numicon and the bar method can help children to visualise this structure.

Overemphasis on Take Away (watch the language)

When young children learn subtraction, the - symbol is often used alongside the words 'take away' which is the case in Year 1 but there are other structures to subtraction that this use of language can interfere with and cause confusion.

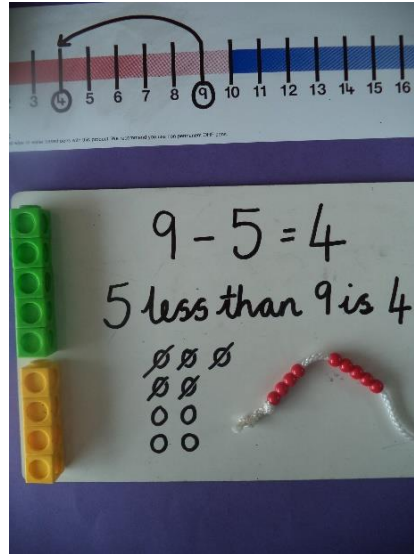
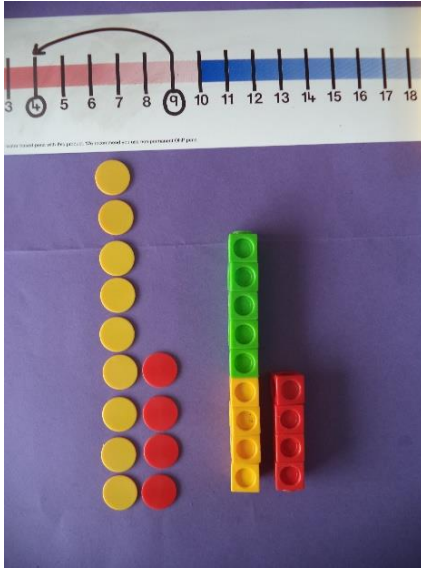
For example, when a child meets '6- (-3)' in temperature, it does not make sense to use the take away structure e.g you can't take away minus 3 from 6. Instead, the comparison structure is being used.

Progression in Subtraction

Year 1

Understand subtraction as taking away: **What is ...less than...?**

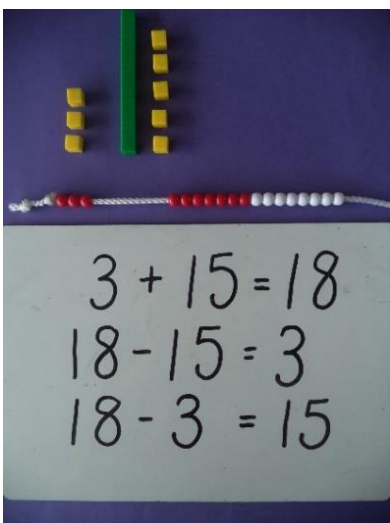
Compare quantities to say how many less and/or how many more



Children may begin recording pictorially progressing to recording number sentences alongside. Use practical resources such as bears, counters, cubes, and numberlines/hundred grids to encourage counting in groups rather than ones

Fluency-mental maths targets for subtraction

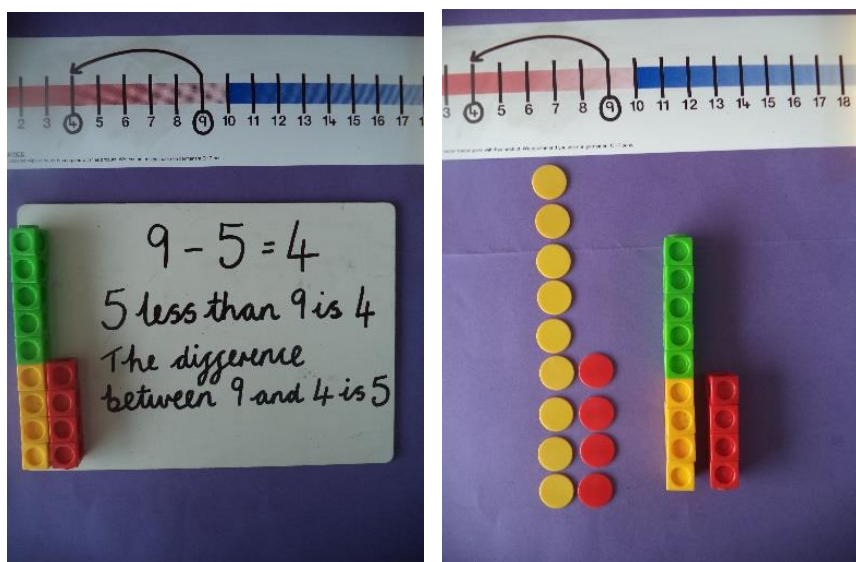
Count backwards (including crossing 100) any given number



- Switch count between ones and tens e.g 33, 32, 31, 30, 20, 10
- Represent and use subtraction facts linked to number bonds up to 20
- Find one less than a number
- Find ten less than a number
- Count back in multiples of 2s, 5s and 10s starting on multiples to highlight pattern

Year 2

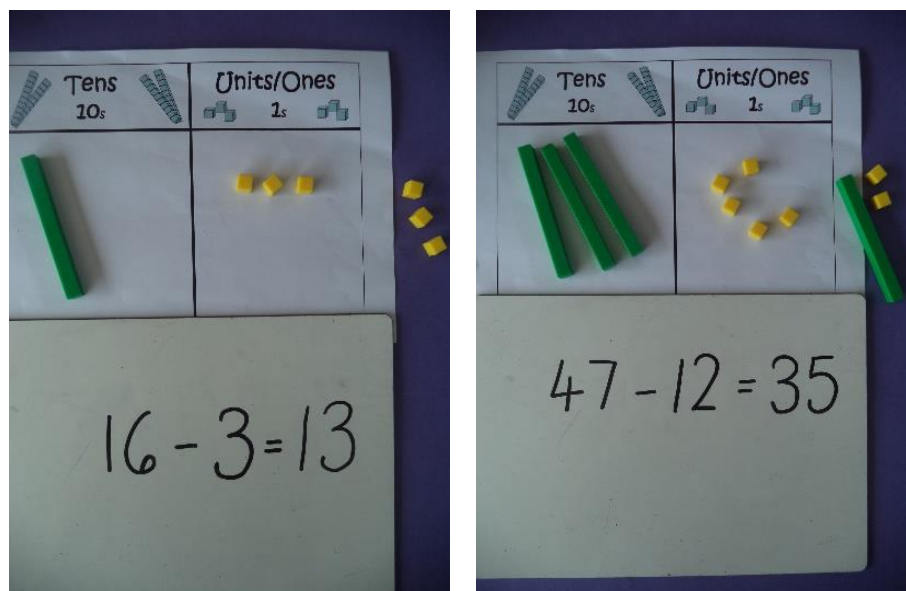
Understand subtraction as taking away and finding the difference



Ensure children understand that subtraction is not commutative (can't be done in any order)

Children should be able to partition numbers in different ways

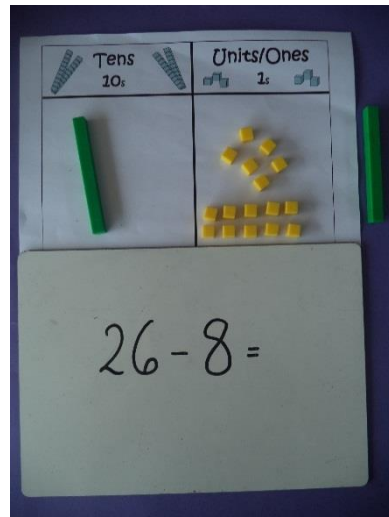
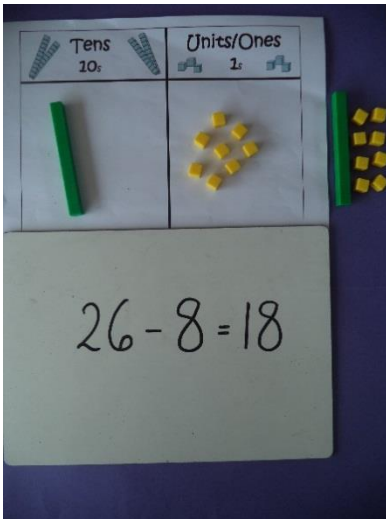
Subtraction no exchanging



Deines are counted out, placed on the place value mat and taken away

Subtraction with exchanging

Step 1 - Deines are counted out and placed on the place value mat

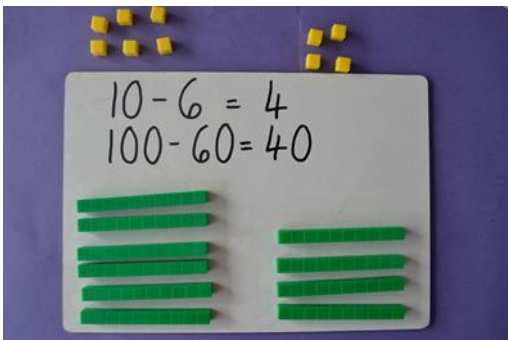


Step 2-One ten is exchanged for ten ones

Step 3-The number is then subtracted

Fluency-mental maths targets for subtraction

- Addition and subtraction facts to 20
- Derive subtraction facts for numbers up to 10 then 20
- Use known facts to 20 to derive new facts e.g $10-7=3$, $100-70=30$

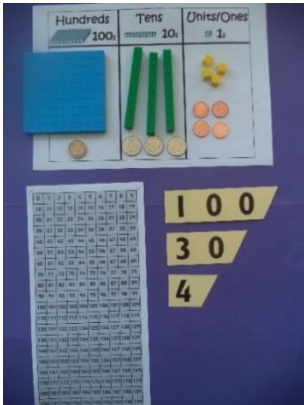


- Derive and use subtraction number facts up to 100

Year 3

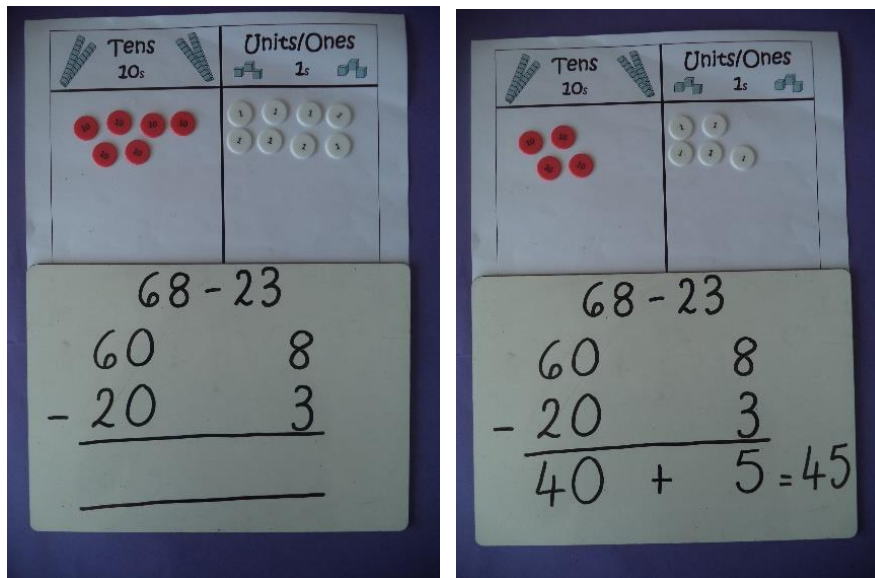
Subtract numbers with up to three-digits (formal written column method)

Children apply, develop and secure their understanding of place value and begin to record in columns

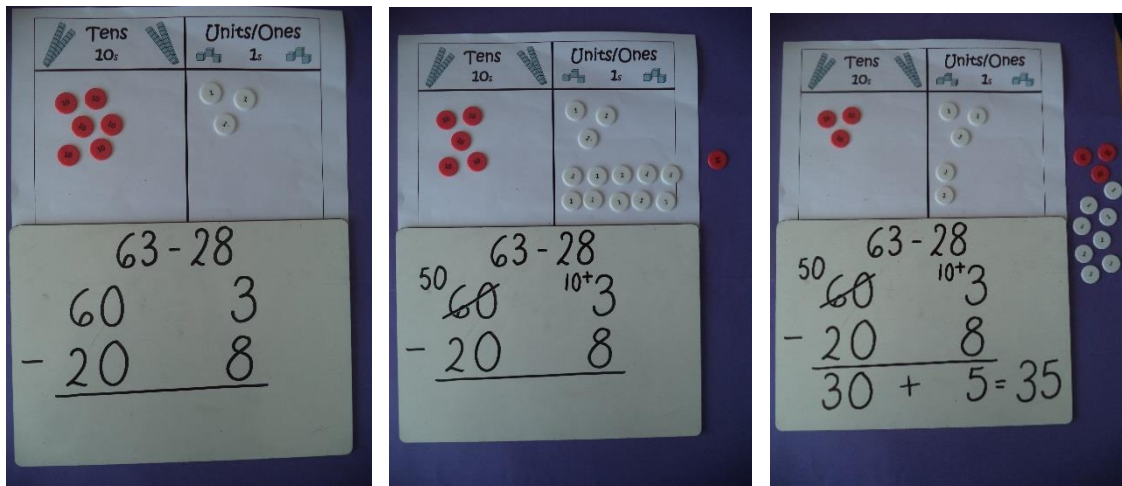


Possible visual and concrete representations

Expanded two-digit subtraction (no exchange)

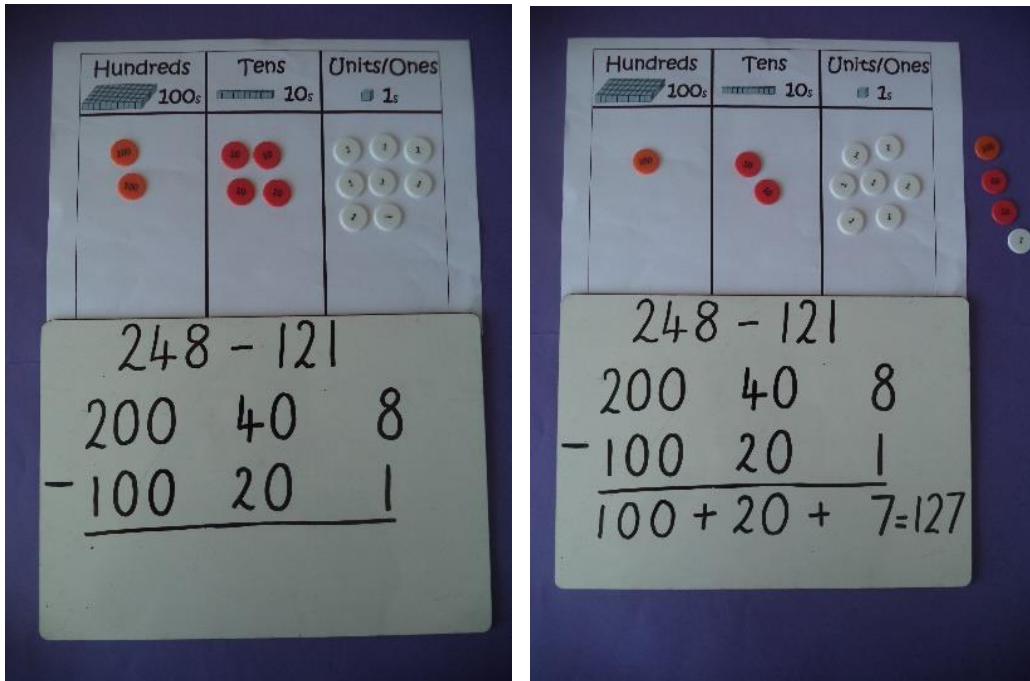


Expanded two-digit subtraction (with exchange)

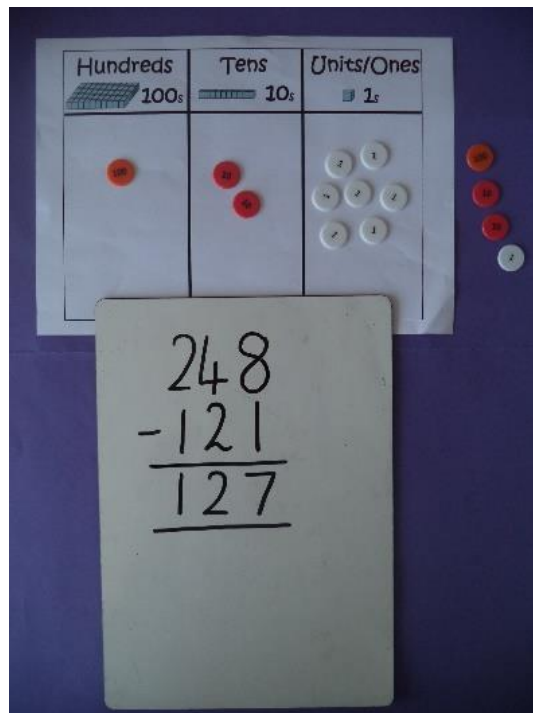


Place value counters are placed on the place value mat alongside the expanded algorithm. A ten counter is exchanged for ten ones.

Expanded three-digit subtraction (no exchange)



Column subtraction (no exchange)



Fluency-mental maths targets for subtraction

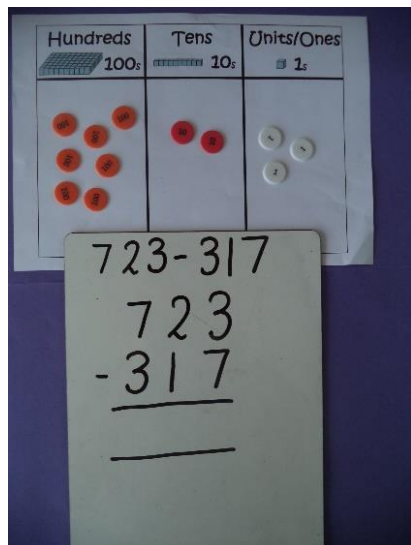
- Count back in ones, tens and hundreds
- Switch count between hundreds, tens and ones e.g 500, 400, 300, 290, 280, 270, 269, 268
- Mentally subtract HTU+ones, HTU+tens, HTU+hundreds
- Perform mental calculations with two-digit numbers
- Find a ten and a hundred less than a number with up to three-digits

Year 4

Subtract numbers with up to four-digits (formal written column method)

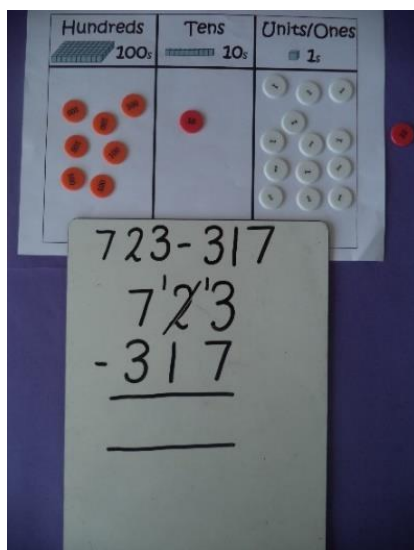
Understand subtraction as the inverse of addition

Solve two-step problems deciding upon the appropriate operations and methods



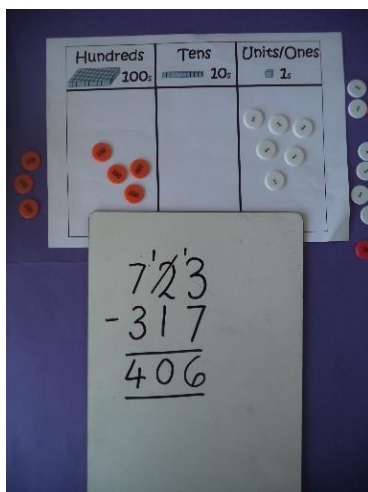
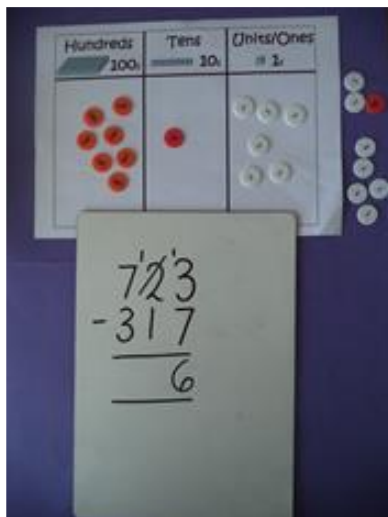
Column subtraction (with exchange)

Step 1 - counters/deines are placed onto the place value mat

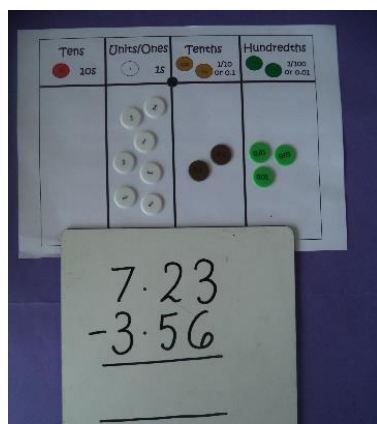


Step 2 - One 'ten' counter is exchanged for ten ones and the process is recorded alongside

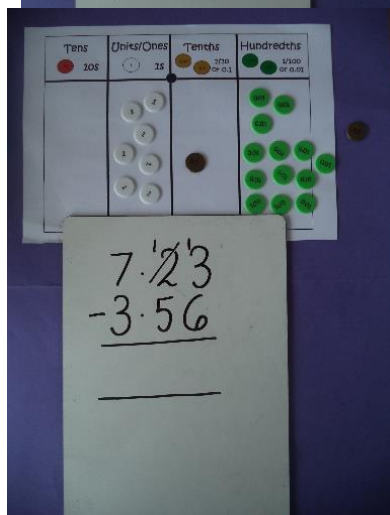
Step 3 - Subtract the ones first, then the tens, then the hundreds



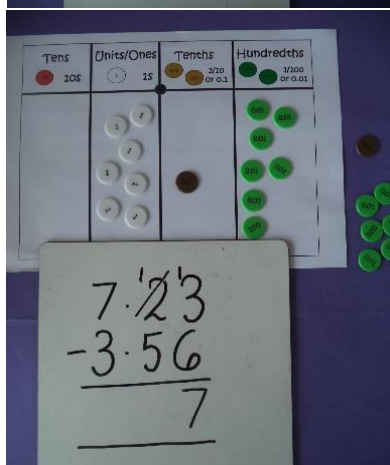
Subtracting decimals using column (with exchange)



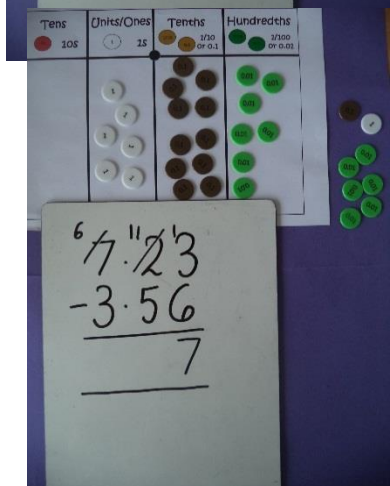
Step 1 - Using the decimal place value mat, count out the correct number of ones, tenths and hundredths



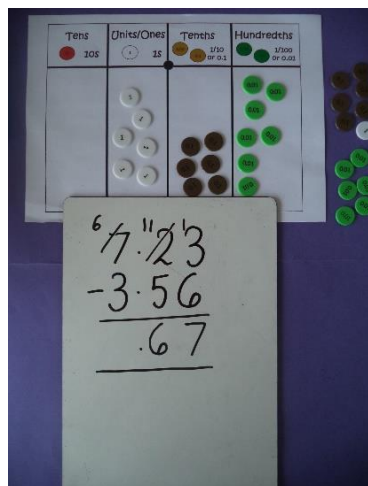
Step 2 - Exchange one tenth for ten hundredths and record the process alongside

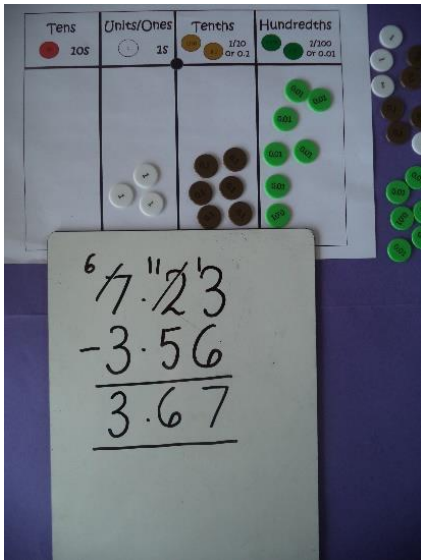


Step 3 - subtract the hundredths



Step 4 - Exchange a one for ten tenths and record the process alongside. Subtract the tenths.





Step 5-Subtract the remaining ones. (Don't forget the decimal point)

Fluency-mental maths targets for subtraction

- Count back in 6, 7, 9, 25 and 1000
- Count back through zero to include negative numbers
- Find 1000 less than a number
- Practice mental calculations with increasingly large numbers

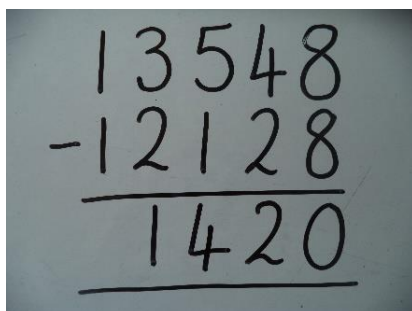
Year 5

Subtract numbers with four-digits or more using formal written column method

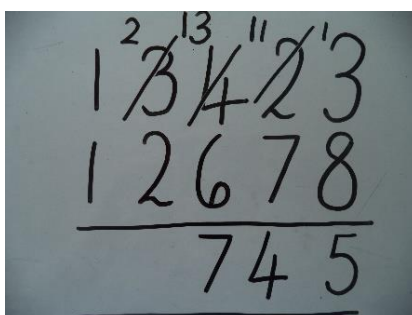
Solve multi-step problems

Subtract numbers mentally with increasingly large numbers

Column subtraction (no exchanging)


$$\begin{array}{r} 13548 \\ -12128 \\ \hline 1420 \end{array}$$

Column subtraction (with exchanging)


$$\begin{array}{r} \overset{2}{1} \overset{13}{\cancel{3}} \overset{11}{4} \overset{1}{\cancel{2}} \overset{1}{3} \\ -12678 \\ \hline 745 \end{array}$$

Ensure children can solve calculations where zero is a place holder

Fluency-mental maths targets for subtraction

- Count backwards in powers of ten up to 1 million
- Count backwards in positive and negative whole numbers through zero
- Practice mental calculations with increasingly large numbers

Year 6

Subtract multi-digit numbers including numbers with up to three decimal places using formal written column method

Solve more complex calculations mentally

Decimal subtraction (no exchanging)

$$\begin{array}{r} 1.48 \\ - 1.21 \\ \hline 0.27 \end{array}$$

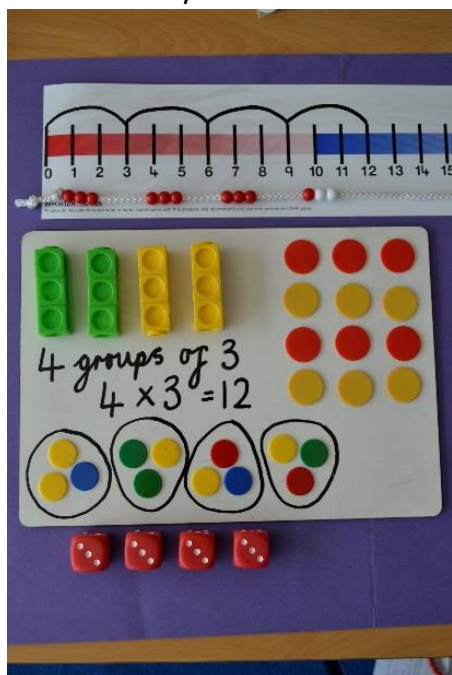
Decimal subtraction (with exchanging)

$$\begin{array}{r} \overset{6}{\cancel{7}}.\overset{11}{\cancel{2}}\overset{1}{3} \\ 3.67 \\ \hline 3.56 \end{array}$$

Multiplication

What is involved in understanding multiplication?

Children need concrete and pictorial experiences when they first meet multiplication and division in order to lay the foundations to which the language and symbols can be connected.



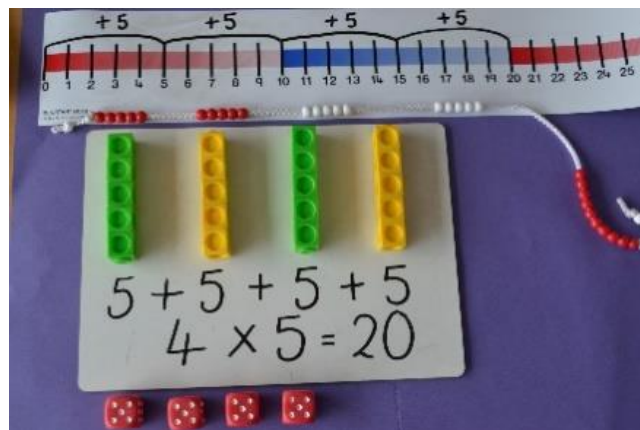
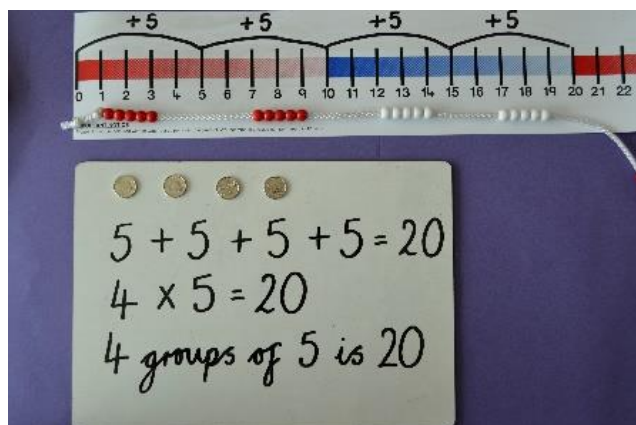
Some possible enactive representations of multiplication (see glossary of resources)

Key Vocab: *times, groups of, lots of, product, multiply, multiplication*

There are 3 structures of multiplication.

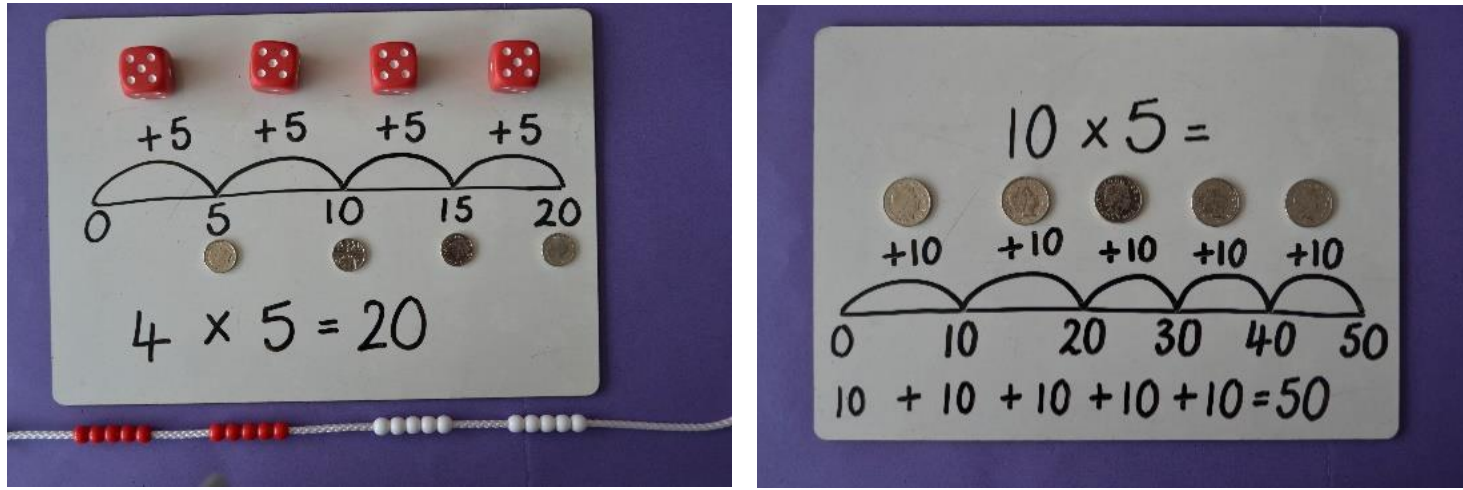
The repeated addition structure of multiplication

Multiplication signs means 'groups of'. We can imagine 4×5 as 4 groups of 5 things. This is the repeated addition structure. Four sets of five can be written as:



The repeated addition of $5+5+5+5=20$ is represented as towers of cubes, on a numberline, dice, using money and on a bead string with the two corresponding number sentences next to them

Begin by using objects familiar to the child and familiar language in the form of stories. Use pictures, arrays and marked number-lines to record, progressing to unmarked.



The empty numberline shows four jumps of 5. The (+5) is written directly above the jump. Beadstrings can be used alongside as an enactive resource. The repeated addition sum is shown. It is important children see the structure of multiplication represented in different ways.

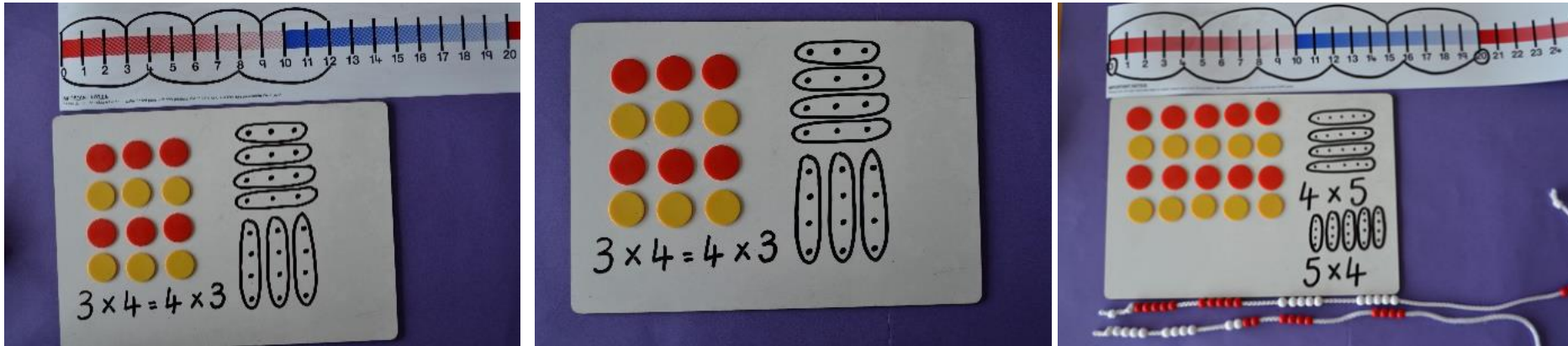
The commutative principle

When two numbers are multiplied together, it does not matter which one comes first. This is like addition. The understanding of this is a hugely significant step in a child's understanding of multiplication. The equivalence between 'four sets of three' and 'three sets of four' is not immediately apparent so it will need to be represented visually to the child in the form of an array using a variety of apparatus/objects.

$$a \times b = b \times a$$

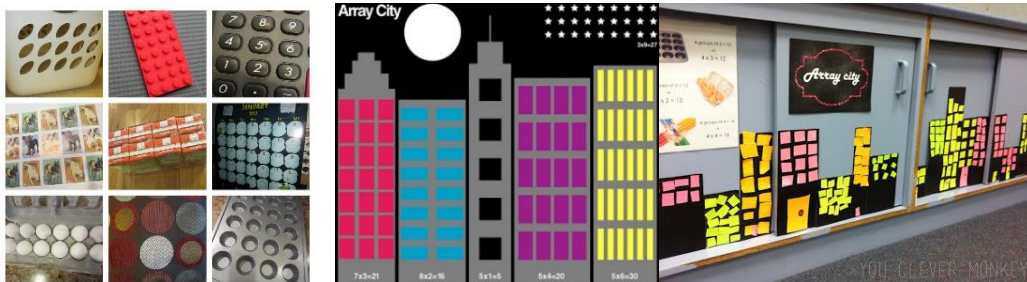
Using arrays

These are essential in establishing the commutative property of multiplication.



The reversible counters are used to create the array enactively before it is represented pictorially on the number line and as dots on the whiteboard. The beadstrings also demonstrate commutativity.

Encourage children to see rectangular arrays in the environment.



Egg boxes, lego, windows, baking trays, classroom tray drawers all have arrays which can be drawn and 'looped' to work out the multiplication fact. The array is represented using apparatus before being represented pictorially and 'looped' to show the two commutative facts.

The scaling structure

Numbers are 'scaled up according to a 'scale factor'. In the example below the number of pens is scaled up by a factor of 9; 'nine times as many'.

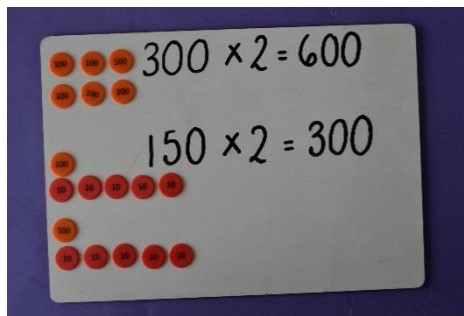
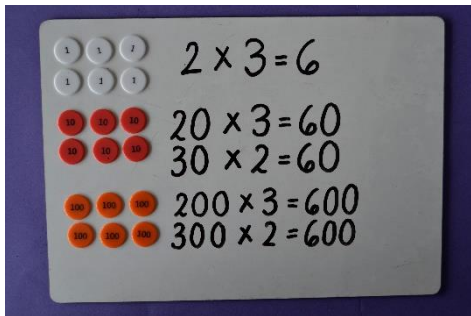
I have three pens. My friend has nine times as many. How many has she got?

Nine times as many pens as three. $9 \times 3 = 27$

Scaling in real-life

- Height-comparing a person to a statue
- Recipes
- Models
- Plants-comparing a bonsai to a tree
- Animals-baby to adult
- Salaries
- Journey times/distances
- Mixing-chemicals, concrete and paint

Using the scaling structure, the children are able to work out other multiplication facts by scaling them up. We call these 'mega tables'.



For example,

$3 \times 2 = 6$

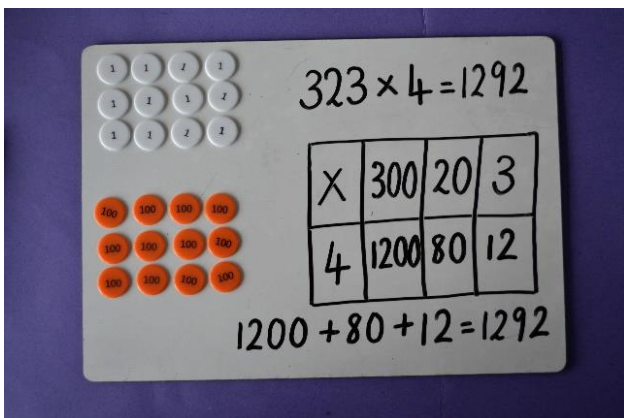
$30 \times 2 = 60$

$30 \times 20 = 600$

$300 \times 2 = 600$

$150 \times 2 = 300$

$5 \times 2 = 30$



The place value counters and array representation are used to scale up by a factor of 10 each time. The product of 300×2 is then halved or scaled down, again using apparatus to represent.

Children will need to be able to do this when multiplying using the grid method and formal recording.

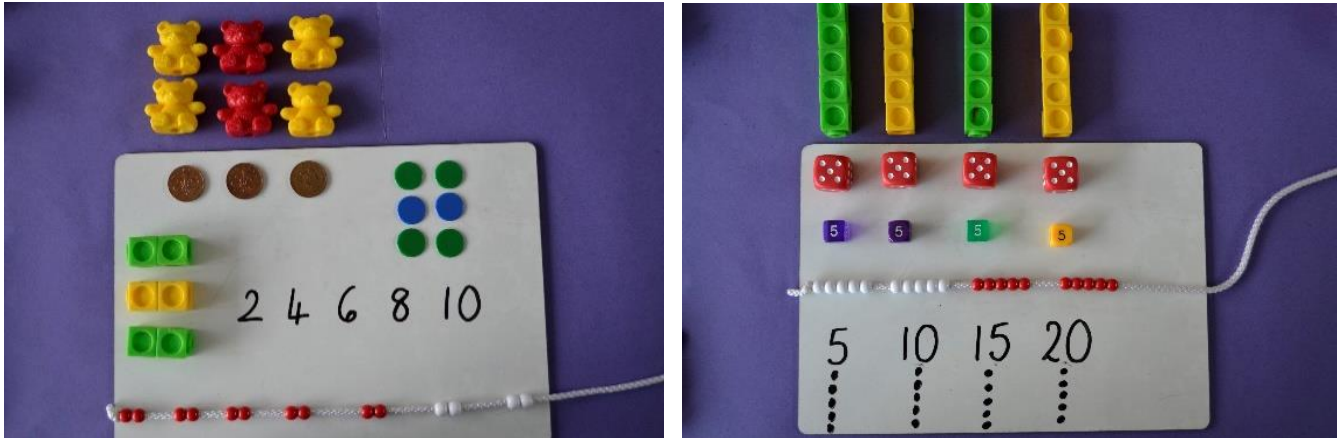
Progression in Multiplication (see glossary for resources)

Year 1

Solve single step practical problems involving multiplication

Use concrete objects, pictorial representations to explore grouping

Make connections between arrays, number patterns and counting in twos, fives and tens



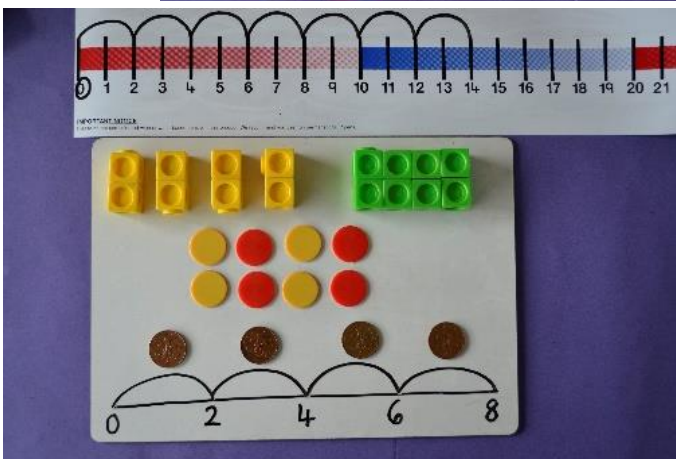
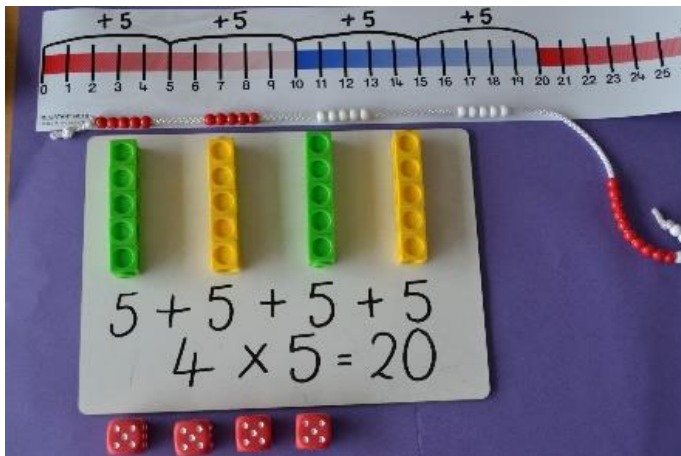
At this stage, exploration of counting patterns is practical only, using concrete experiences, pictorial representations and arrays.

Fluency-mental maths targets for multiplication

- Count in twos, fives and tens from different multiples e.g 6, 8, 10, 12
- Emphasise number patterns
- Double number and quantities

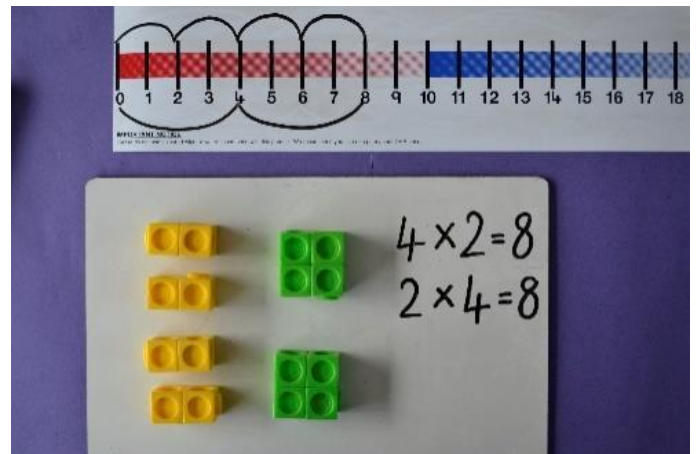
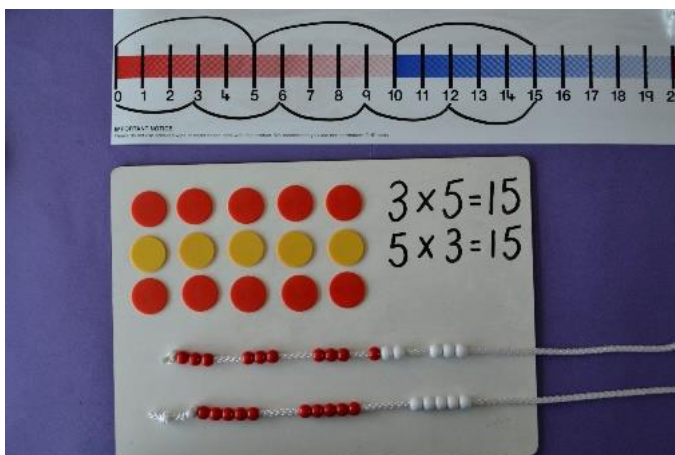
Year 2

Understand multiplication as repeated addition

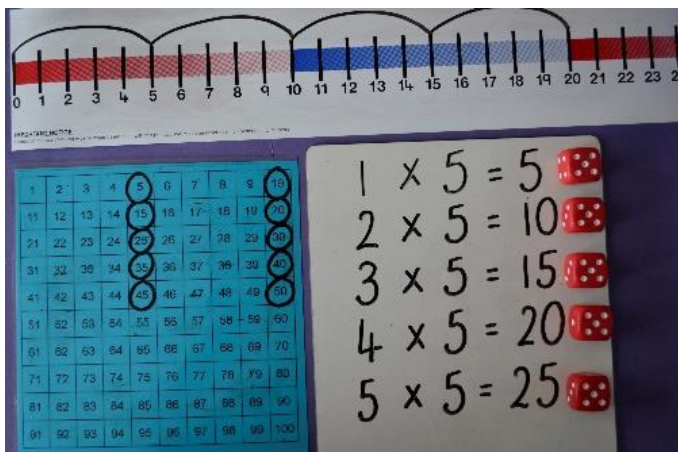


Connect concrete representations with marked and unmarked numberlines to show the repeated addition structure.

Calculate mathematical statements for multiplication within the tables and write them using symbols
Understand and solve problems involving arrays Ensure children understand that multiplication is commutative



Understand that multiplication and division are inverse operations



Connect concrete representations such as arrays, cube towers and beadstrings with pictorial representations such as drawn arrays and numberlines to show the commutative property.

Emphasis patterns with numberlines and hundred squares so that children begin to recognise when a number is a multiple of 2, 5 or 10.

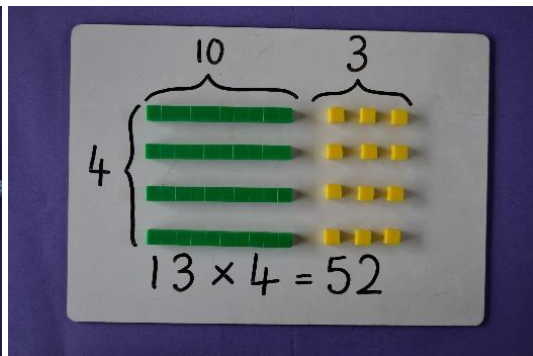
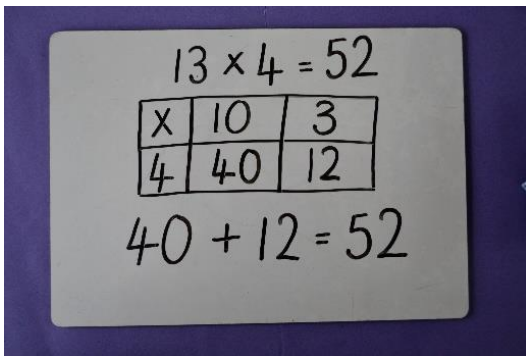
Fluency-mental maths targets for multiplication

- Count in twos, threes and fives from zero and tens from any number
- Introduction to multiplication tables. Practice to become fluent in multiplication facts for 2, 5 and 10
- Solve multiplication problems mentally e.g There are 5 pencils in a pot. How many pencils will be in 4 pots?

Children will need practical exploration in order to commit times table facts to memory.

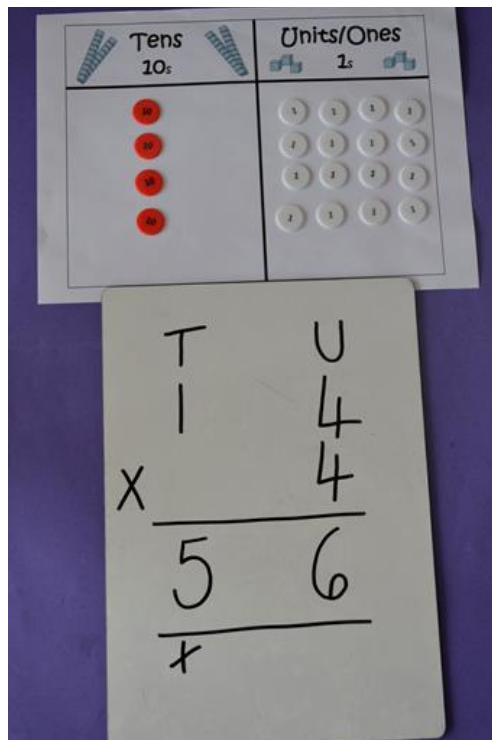
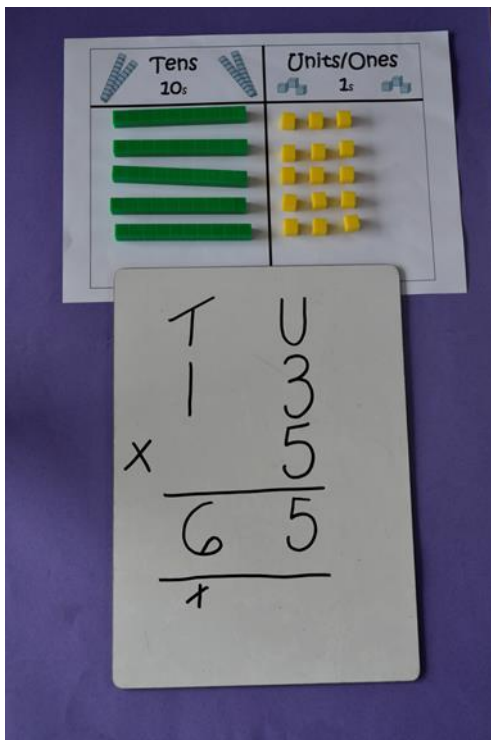
Year 3

Develop reliable written methods such as the grid method moving towards short multiplication



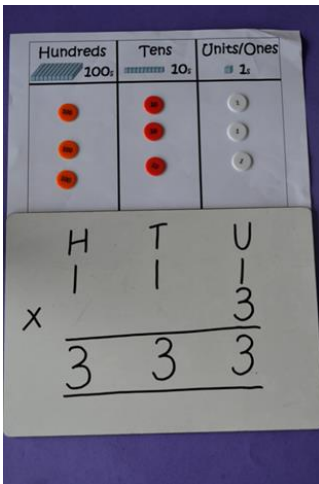
**Manipulatives
MUST be used
alongside
algorithms**

Understand and solve scaling problems. Solve problems involving multiplication, including correspondence. Use deines to give children concrete experiences of multiplying past 12 times to help children appreciate the place value.

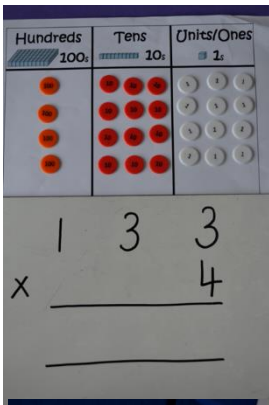


Count out deines or place value counters onto the place value mat alongside the written calculations

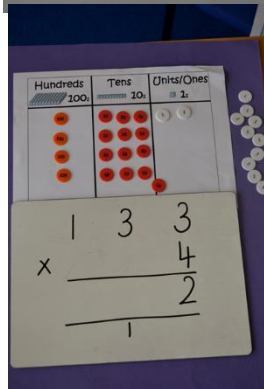
Multiplication (no exchanging)



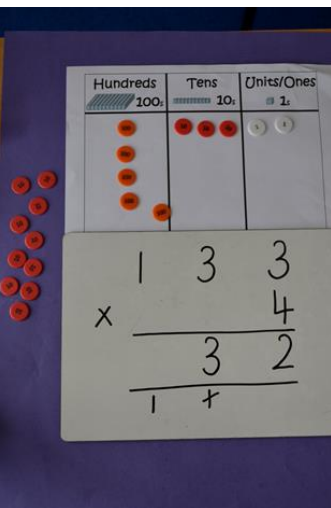
Multiplication with exchanging



Step 1-Count out place value counters and place onto place value mat. Here you can see 133 arranged into 4 rows.

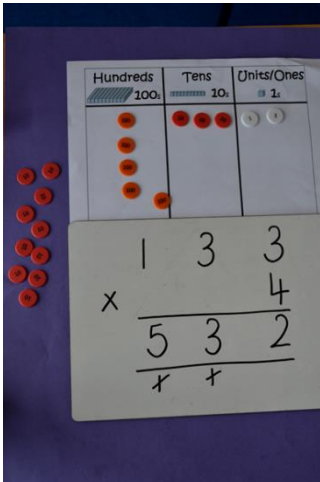


Step 2-Exchange ten place value counter 'ones' for a ten and write correct notation. 3 groups of 4 counters made 12 (10 and 2 ones) which has been written below.

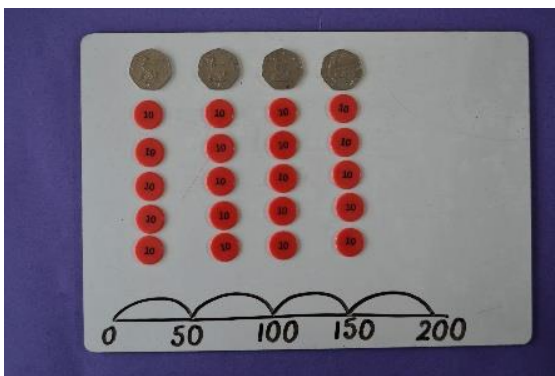


Step 3-Exchange ten 'ten' counters for a hundred counter and write correct notation. This will help children to appreciate the algorithm and notation. The exchanged 10 and 1 is crossed off when it has been used.

Step 4 Complete the calculation

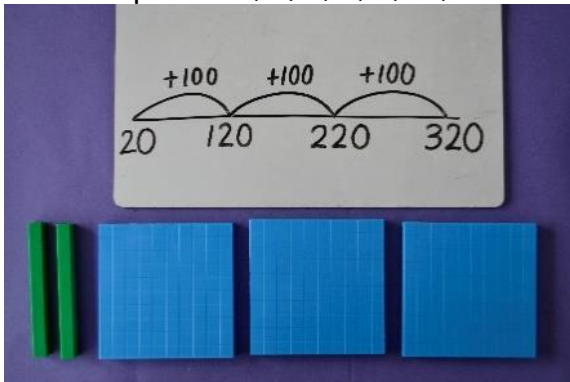


Fluency-mental maths targets for multiplication



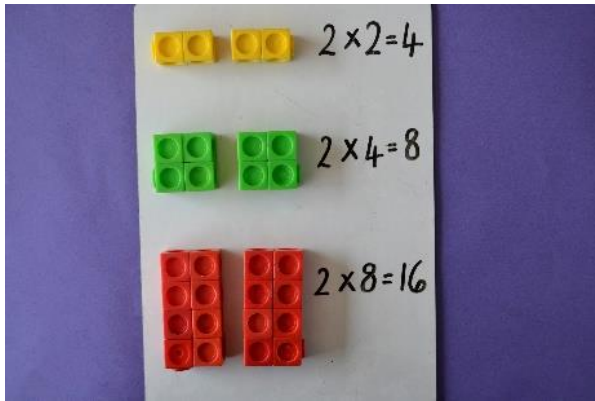
- Count from 0 in multiples of 4, 8, 50 and 100
- Counting in 50s represented as money and using place value counters with empty numberline recording alongside.

- Use multiples of 2, 3, 4, 5, 8, 10, 50 and 100



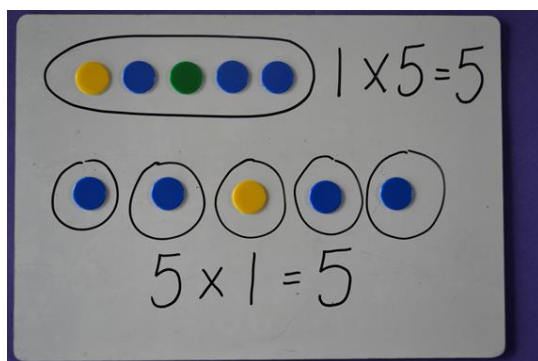
Daily counting using the place value chart will help with fluency

- Practice mental recall of multiplication tables-3, 4 and 8 times tables



- Connect the 2, 4 and 8 times tables using doubling
Use resources such as cubes to help children to visualise this
- Develop efficient mental methods using commutativity and multiplication facts to derive related facts e.g $12 \times 4 \times 5 = 12 \times 20$

Year 4



Multiplying three numbers

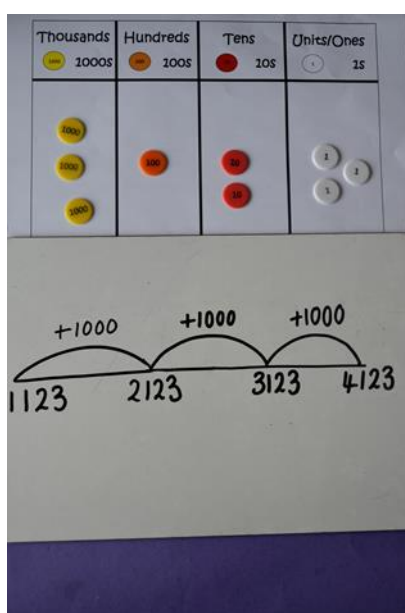
Solve two-step problems

Multiplying by 0 and 1. Use apparatus to help children visualise. A common mistake is multiplying by 0 and getting a whole number product.

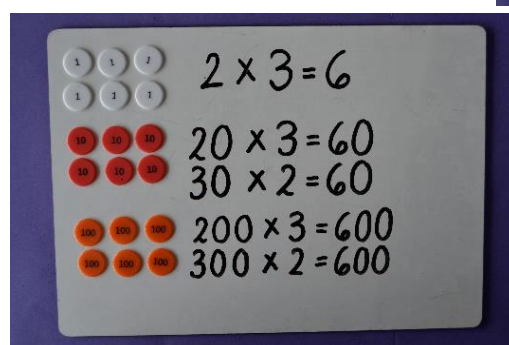
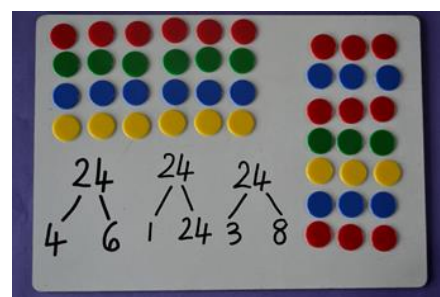
Develop fluency in short multiplication using formal written layout, see Year 3 for a detailed explanation.

Solve problems involving multiplication using distributive law, integer scaling problems and harder correspondence problems

Fluency-mental maths targets for multiplication



- Count in multiples of 6, 7, 8, 9, 25 and 1000
- Recall and use multiplication facts up to 12×12
- Derive multiplication facts with up to three-digits



- Recognise and use factor pairs and commutativity in mental calculations
- Use the distributive law
- Combine knowledge of number facts and rules of arithmetic to solve mental and written calculations e.g $2 \times 6 \times 5 = 10 \times 5$

Year 5

Multiply 4 digit numbers by whole numbers

A place value chart with columns: Thousands (1000s), Hundreds (100s), Tens (10s), and Units/Ones (1s). Below the chart, the multiplication 1232×3 is written and solved to get 3696.

Thousands	Hundreds	Tens	Units/Ones
1000s	100s	10s	1s
1000	200	30	2
1000	200	30	2
1000	200	30	2
			3
<hr/>			
3	6	9	6

A place value chart with columns: Tens (10s), Units/Ones (1s), Tenths (1/10 or 0.1), and Hundredths (1/100 or 0.01). Below the chart, the multiplication 2.23×3 is written and solved to get 6.69.

Tens	Units/Ones	Tenths	Hundredths
10s	1s	1/10 or 0.1	1/100 or 0.01
	2	2	3
	2	2	3
	2	2	3
			3
<hr/>			
	6	6	9

A factor tree diagram showing 12 and 24 factored into 2 and 6. To the left, there are 12 green counters and 24 red counters arranged in groups.

Use the decimal place mat with place value counters alongside teaching the formal algorithm if needed

Identify all multiples and factors including finding all factor pairs of a number, and common factors of two numbers

Solve problems involving all four operations

Multiply whole numbers and those involving decimals by 10, 100 and 1000

A place value mat with columns: Hundreds, Tens, Ones, Tenths, Hundredths, Thousandths. The number 5.31 is placed on the mat. Below it, the equation $5.31 \times 10 =$ is written.

A place value mat with columns: Hundreds, Tens, Ones, Tenths, Hundredths, Thousandths. The number 53.1 is placed on the mat. Below it, the equation $5.31 \times 10 = 53.1$ is written.

Emphasis that the decimal point does not move but the place value of the digits.

Place value chart showing the conversion of 4.4 km to meters. The chart has columns for Tens, Ones, Tenths, and Hundredths. The number 4.4 is written across these columns. Below the chart, a whiteboard shows the following equations:

$$1 \text{ km} = 1000 \text{ m}$$

$$4.4 \text{ km} =$$

Place value chart showing the conversion of 4.4 km to meters. The chart has columns for Thousands, Hundreds, Tens, Ones, Tenths, and Hundredths. The number 4,400 is written across these columns. Below the chart, a whiteboard shows the following equations:

$$1 \text{ km} = 1000 \text{ m}$$

$$4.4 \text{ km} = 4,400 \text{ m}$$

Link this concept to money and measures

A whiteboard showing the equation:

$$61 \times 24 = 61 \times \square \times 6$$

Understand and use multiplication as inverses including in problems involving missing numbers and balancing equations

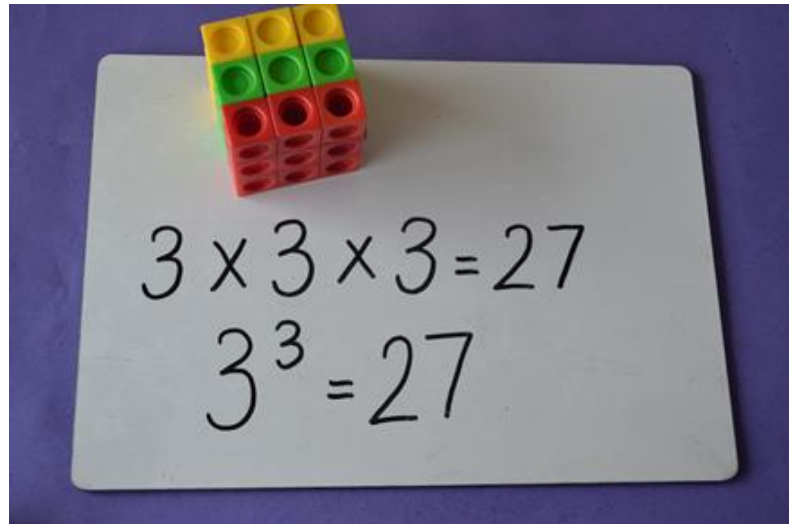
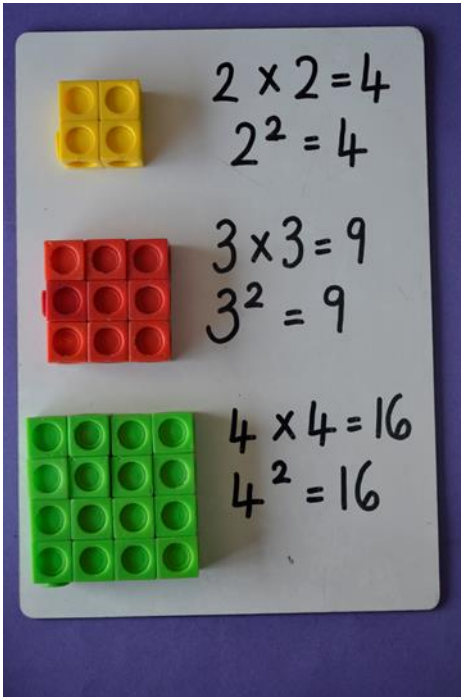
Solve problems involving multiplication and division including scaling by simple fractions

A chart showing prime factors of numbers 1 to 50. The numbers are arranged in a grid. Below the grid, a whiteboard lists the prime factors for the numbers 2, 3, 5, and 7, each represented by a row of colored dots:

- 2: 2 red dots
- 3: 3 blue dots
- 5: 5 yellow dots
- 7: 7 green dots

Know and use the vocabulary of prime numbers, prime factors and composite (non-prime)

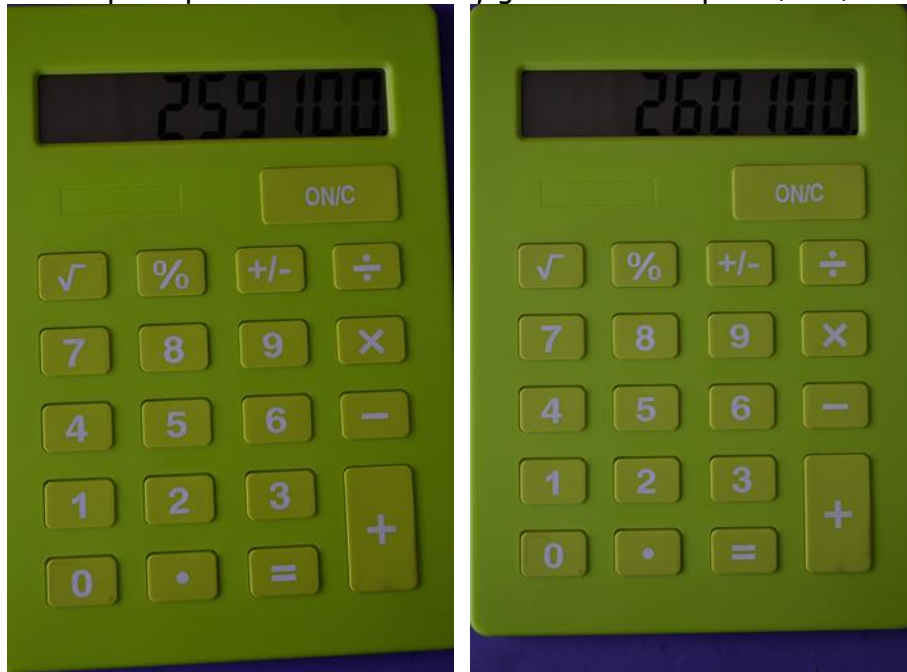
Recognise square and cube numbers and associated notation.



Investigate practically using cubes

Fluency-mental maths targets for multiplication

- Count forwards in steps of powers of 10 from any given number up to 1,000,000



Use the large and small calculators to investigate counting in powers of 10. Discuss which digits stay the same and which ones change.

$$\begin{array}{r}
 \text{Th} \\
 1324 \\
 \times \quad \quad \quad 6 \\
 \hline
 7944 \\
 \hline
 \times \quad \times \quad \cancel{2}
 \end{array}$$

$$\begin{array}{r}
 3.24 \\
 \times \quad \quad \quad 6 \\
 \hline
 19.44 \\
 \hline
 \times \quad \cancel{2}
 \end{array}$$

- Practise and extend use of formal written method of short multiplication
- Apply all multiplication tables frequently. Commit them to memory and use them confidently to solve larger calculations.
- Multiply numbers mentally drawing upon known facts

Year 6

Multiply numbers up to 4 digit x TU

$$\begin{array}{r} 1324 \\ \times \quad 2\textcircled{6} \\ \hline 7944 \\ \text{+} \quad \text{+} \quad \text{+} \end{array}$$

Step 1-Multiply the number by the units digit first.

$$\begin{array}{r} 1324 \\ \times \quad \textcircled{2}6 \\ \hline 7944 \\ \text{+} \quad \text{+} \quad \text{+} \quad 0 \end{array}$$

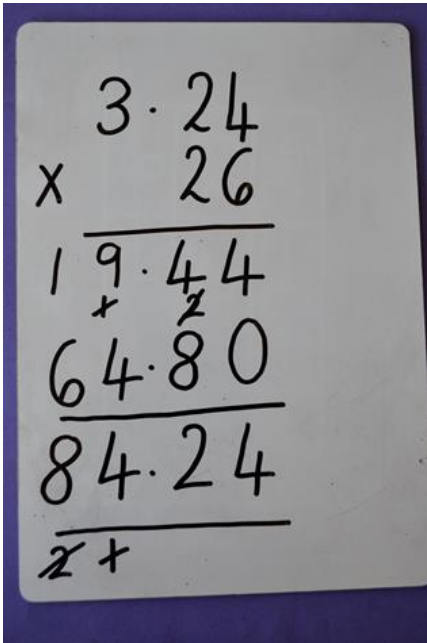
Step 2-Write a 0 underneath the units place before multiplying the number by the tens digit.

Step 3-Add the two answers together.

$$\begin{array}{r} 1324 \\ \times \quad \textcircled{2}6 \\ \hline 7944 \\ \text{+} \quad \text{+} \quad \text{+} \\ 26480 \\ \hline \hline \end{array}$$

$$\begin{array}{r} 1324 \\ \times \quad \textcircled{2}6 \\ \hline 7944 \\ \text{+} \quad \text{+} \quad \text{+} \\ 26480 \\ \hline 34,424 \\ \text{+} \quad \text{+} \quad \text{+} \end{array}$$

Multiply numbers with up to two decimal places x whole number



Multiply multi-digit numbers up to four-digits by a two digit whole number

Multiply single-digit numbers with up to two decimal places by whole numbers

Solve multiplication problems

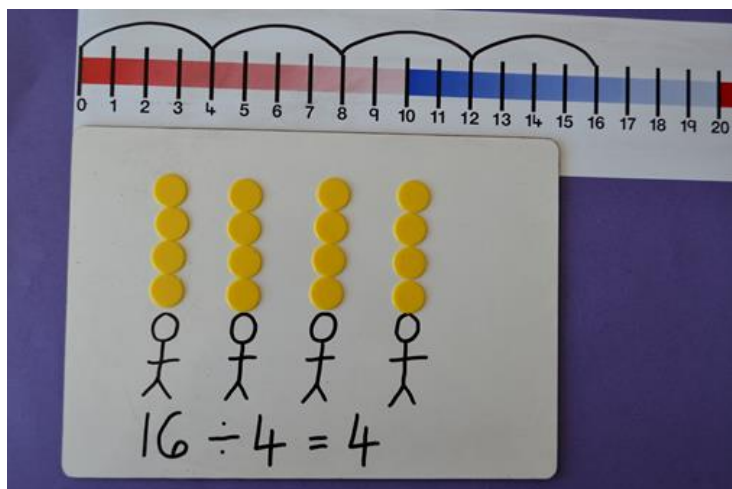
Fluency-mental maths targets for multiplication

- Undertake mental calculations with increasingly large numbers
- Continue to use all multiplication tables to calculate mathematical statements in order to maintain fluency

Division

What is involved in understanding division?

There are two main structures for division: **equal sharing** and **inverse of multiplication**. The division symbol can represent the idea of **sharing** and of **grouping**.



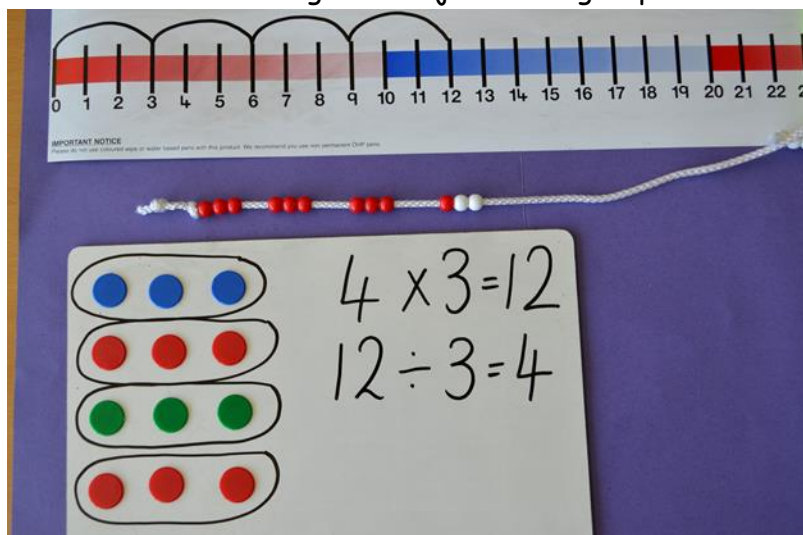
$16 \div 4 = 4$ is the amount each person gets if 16 items are shared equally among four people. The answer is the number in each group.

This is the equal sharing structure

The inverse of multiplication structure

You could also interpret the division number sentence as 'How many threes make 12?'

You would then arrange the objects into groups of 3 and the number of groups would be the answer.



This is the *inverse of multiplication structure*.

This structure is very important as children can then use their times tables to work out division facts which is an important concept to understand.

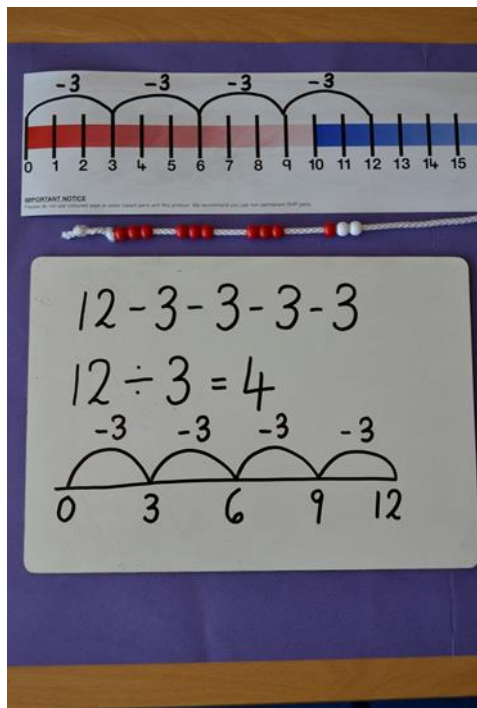
When introducing division to children we often use the **sharing structure** with children as we know that this is a familiar

situation **but** this is a **limited structure** and of less use than the **inverse structure**.

When children meet $6 \div 0.25$ you could not physically share 6 between 0.25 of a person! It would be more sensible to say 'how many amounts of 0.25 make 6?'

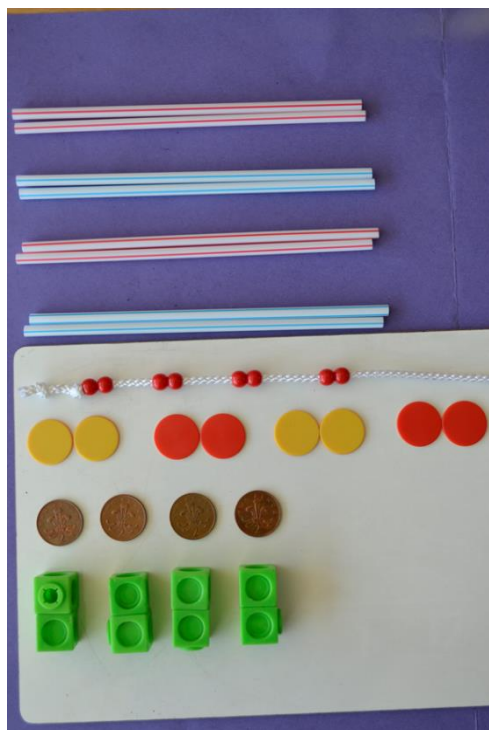
Repeated Subtraction Structure

As with the multiplication repeated addition structure, the repeated subtraction can be represented using a variety of resources before being represented on marked and then unmarked numberlines.



Progression in Division (see glossary for resources)

Year 1



numbers and quantities

Solve single step practical problems involving division

Use concrete objects, pictorial representations

Understand division as grouping and sharing

Use the language of 'sharing equally between'

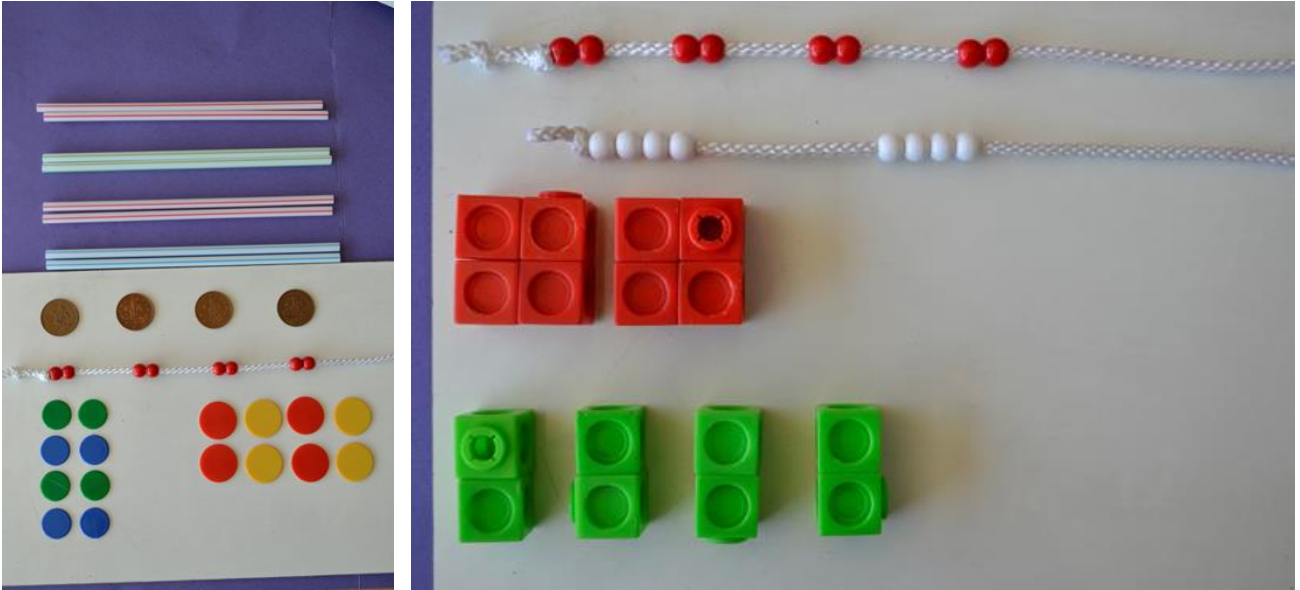
Fluency-mental maths targets for division

- Count in twos, fives and tens from different multiples
- Emphasise patterns
- Find simple fractions e.g half and quarter, of objects,

Year 2

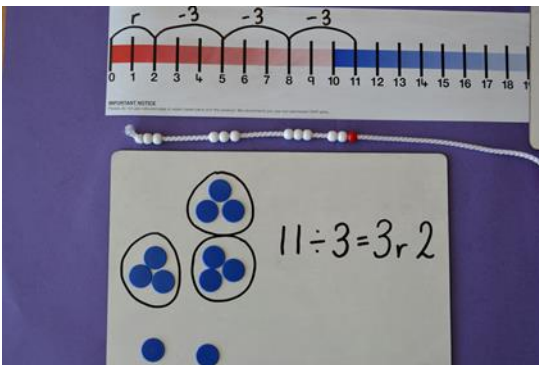
Solve single step practical problems involving division

Use concrete objects, pictorial representations



Use discrete objects such as toys, cubes, reversible counters, the bead string and straws to explore grouping and sharing.

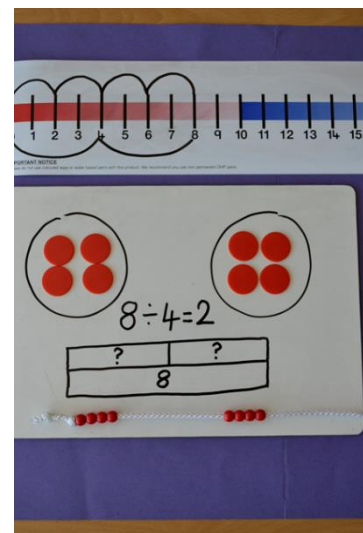
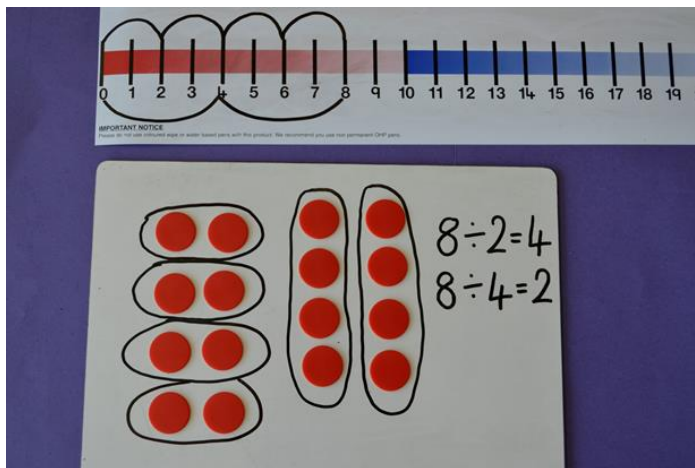
Explore division as grouping including remainders





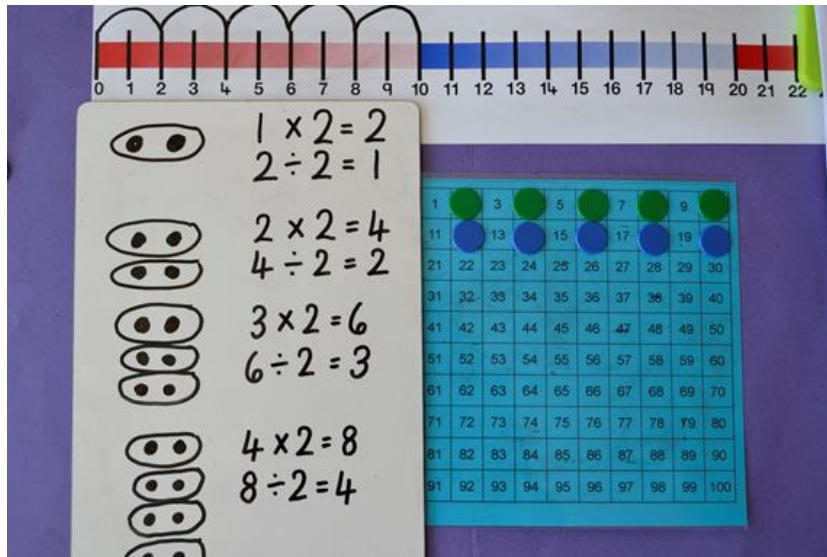
Find halves and then quarters

Work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete quantities and to arrays

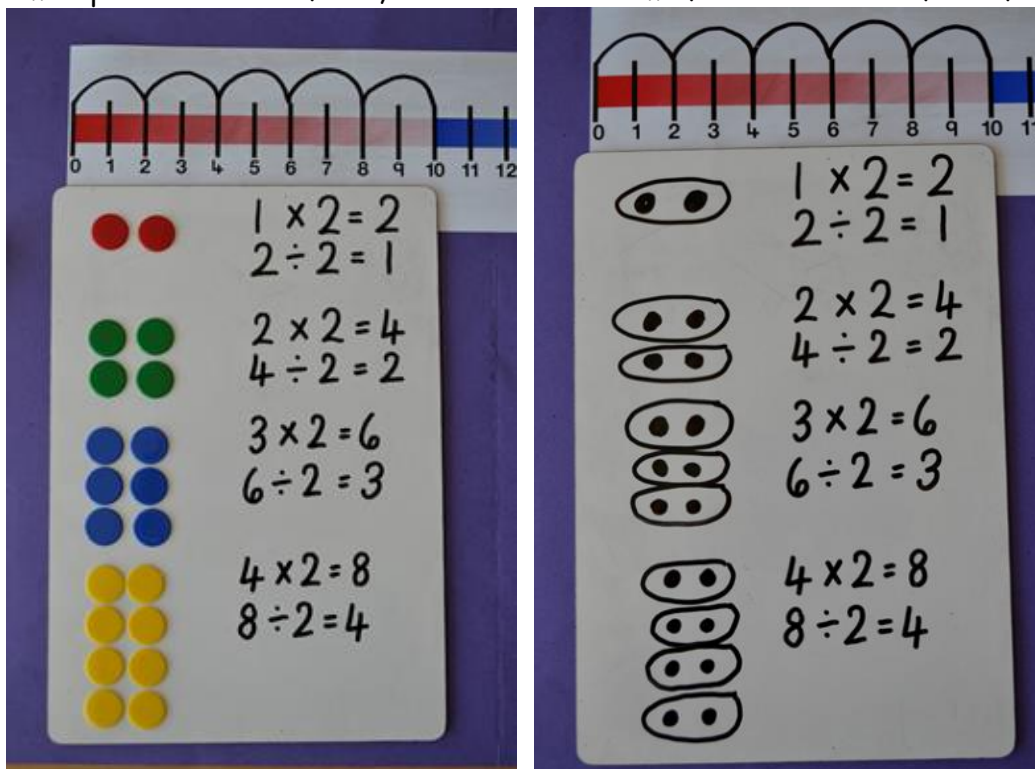


Fluency-mental maths targets for division

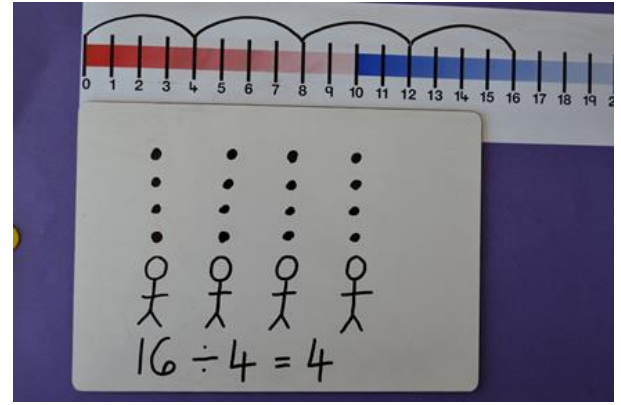
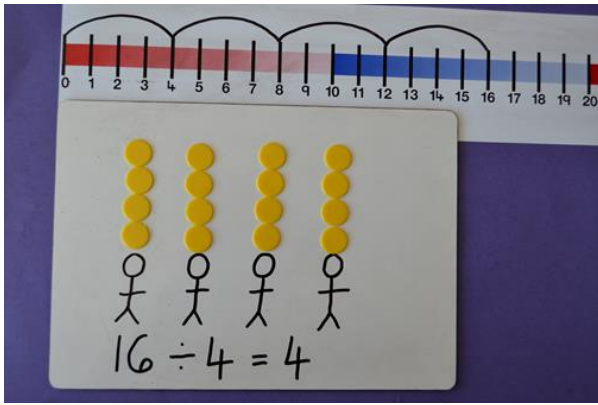
- Count back in twos, threes, fives from zero and tens from any number
- Emphasise patterns



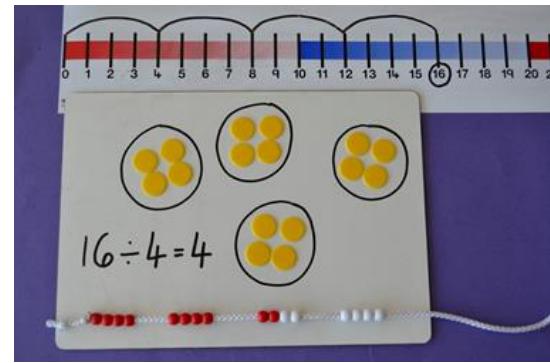
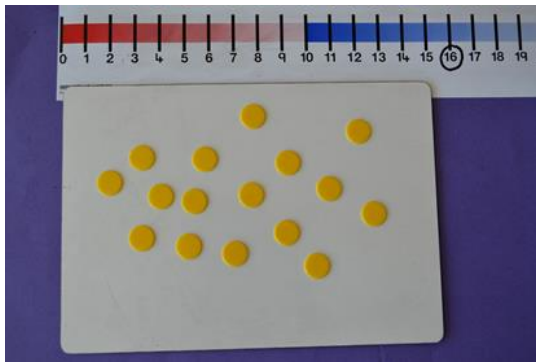
- Connect ten times table to place value and five times table to divisions on a clock face
- Introduce multiplication tables for 2, 5 and 10 and become fluent in division facts for these



- Solve division problems involving grouping and sharing e.g I have 16 sweets and I share them among my 4 friends. How many does each friend get? I have 16 marbles and I put them into bags of 4. How many bags will I need?



Solve problems with enactive then pictorial representations

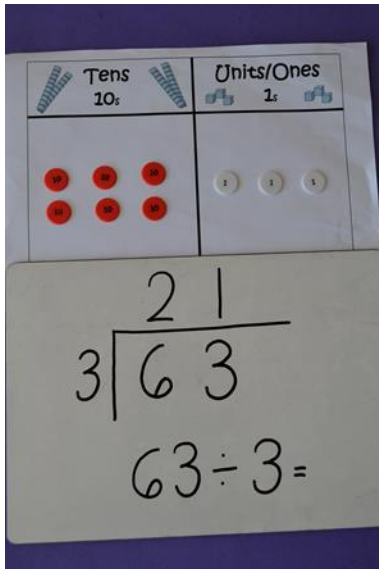


The top pictures show the sharing structure and the pictures above this writing show the grouping structure. The children need to recognise when they are sharing and when they are grouping when solving problems.

Year 3

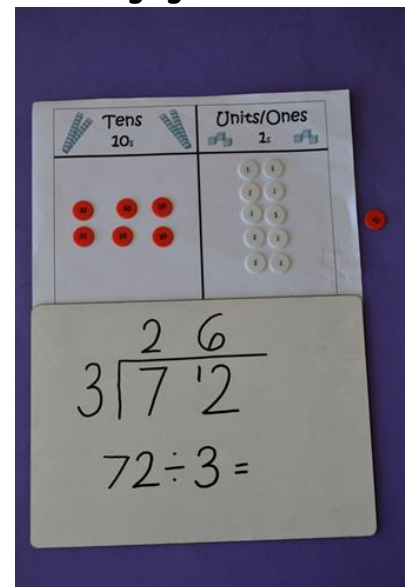
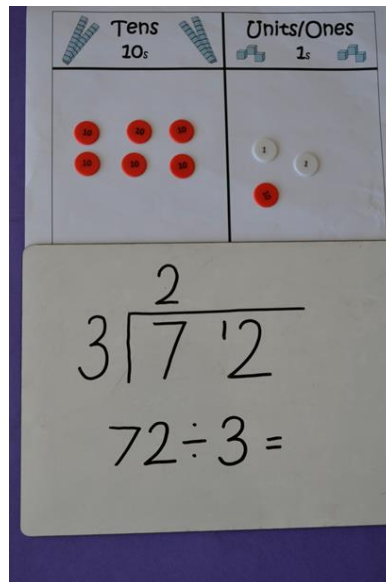
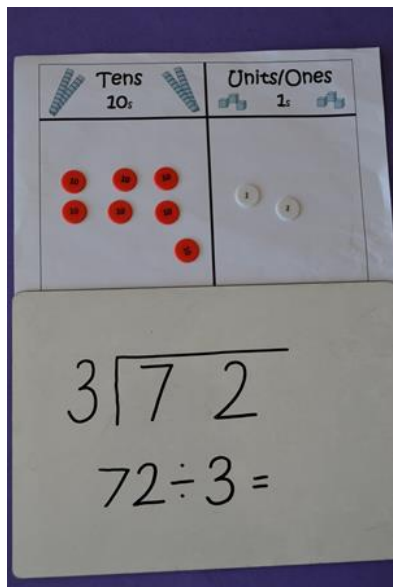
Develop a reliable written method for division

Bus stop method no exchanging



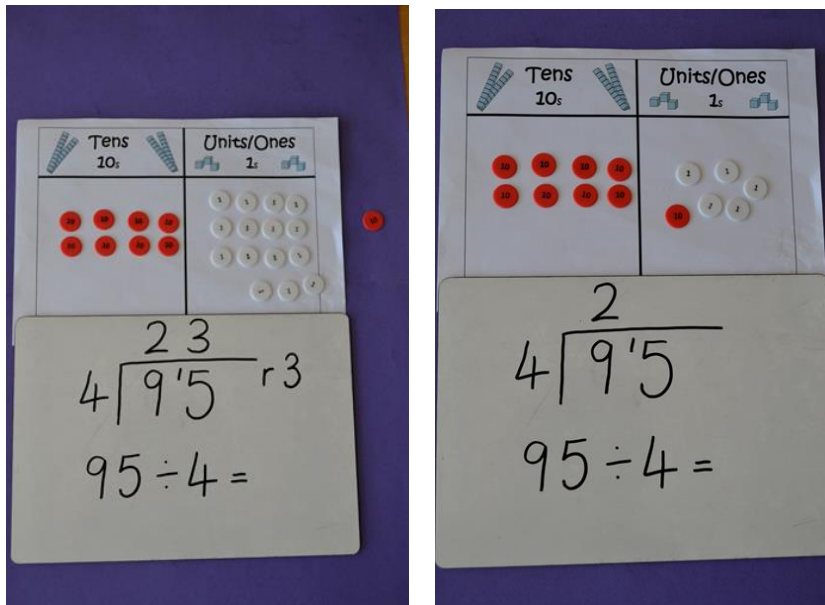
Place value counters are used alongside the written calculation and grouped.

Bus stop with exchanging



Place value counters are grouped into 3s and then the remaining ten is exchanged for ten ones.

By using the manipulatives, the children appreciate why the algorithm is written in this manner.



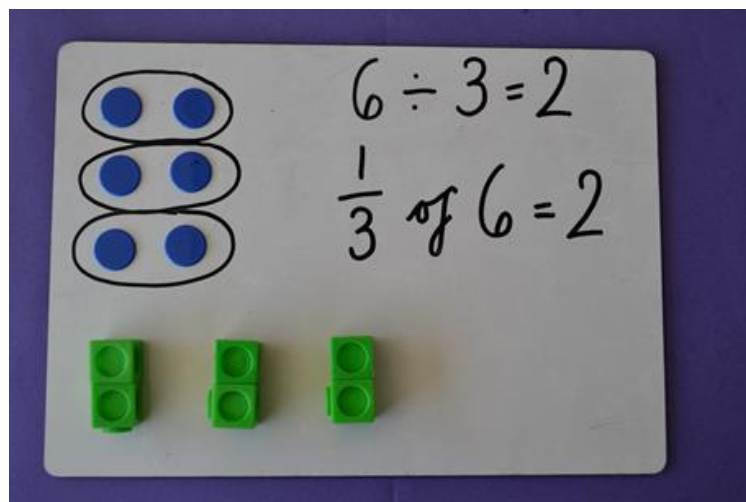
In this example, there is remainder.

Solve problems involving missing numbers

Solve problems involving scaling

Recognise, find and name $\frac{1}{2}$ or $\frac{1}{4}$ of an object, shape or quantity

Understand the link between unit fractions and division

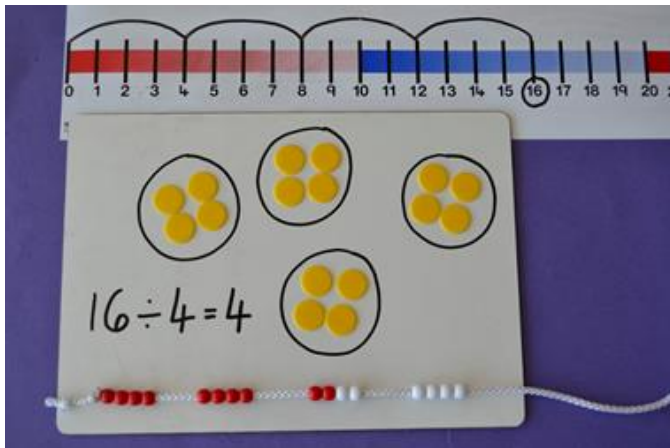


Connect $\frac{1}{10}$ to division by 10

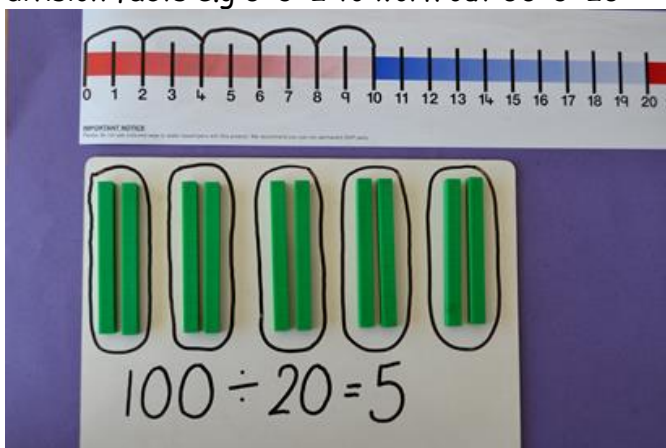
Count in tenths

Fluency-mental maths targets for division

- Recall and use related division facts for the 3, 4 and 8x tables

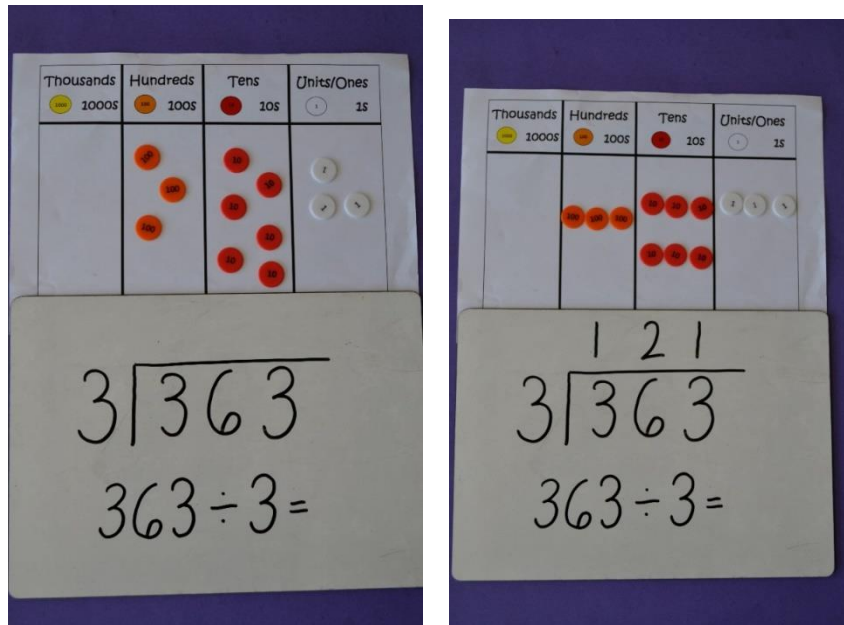


- Write and calculate mathematical statements for division Use division facts to derive related division facts e.g $6 \div 3 = 2$ to work out $60 \div 3 = 20$



Year 4

Become fluent in the formal written method of short division with exact answers when dividing by a one-digit number



Divide one or two-digit numbers by 10 or 100, identifying value of digits as tenths or hundredths



Emphasise that the decimal point does not move and it is the place value of the digits that move by a scale factor of 10



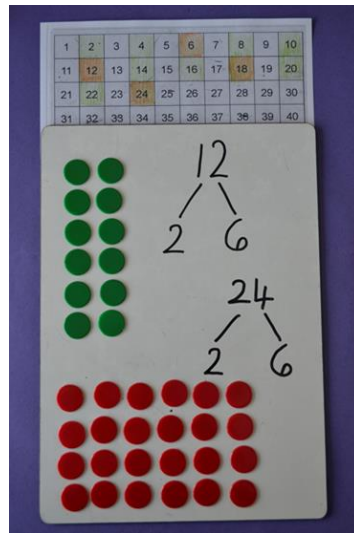
Solve two-step problems in different contexts e.g three cakes shared equally between 10 children

Fluency-mental maths targets for division

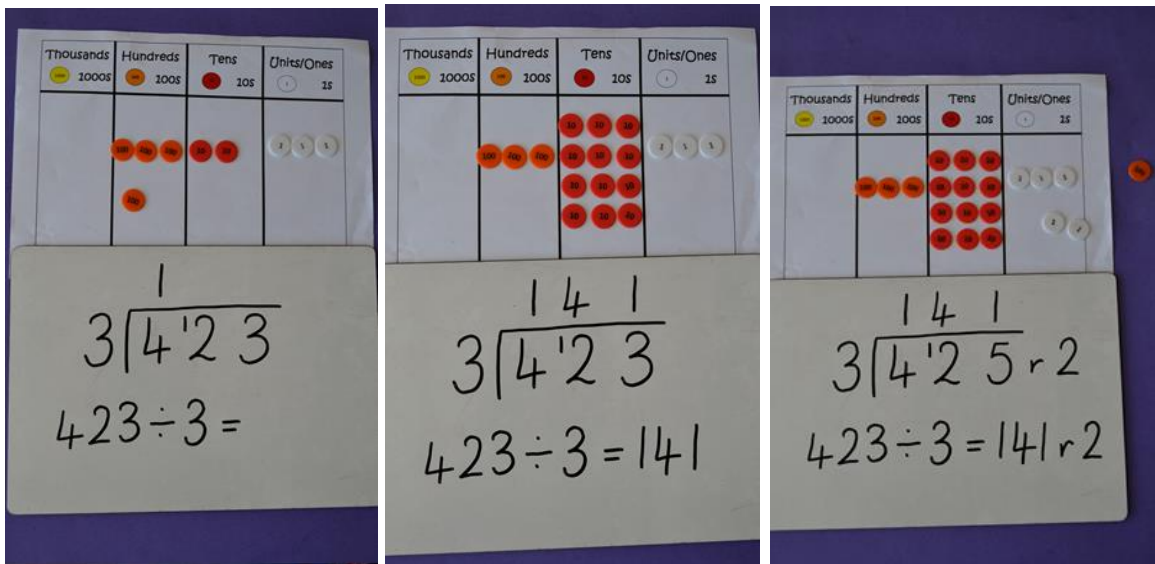
- Continue to practice recalling division facts for multiplication tables up to 12 x 12
- Practise mental methods and extend this to three-digit numbers for example $200 \times 3 = 600$
 $600 \div 3 = 200$
- Use place value, known and derived facts to divide mentally, including dividing by 1
- Recognise and use factor pairs and commutativity in mental calculations

Year 5

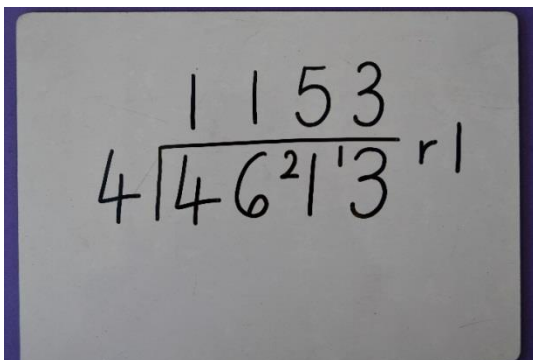
Identify factors, including finding all factor pairs of a number



Practise and extend the formal written method of short division: numbers up to four digits by a one digit number



The place value counters can be used alongside the written calculation if needed. In the above example, the place value counters have been exchanged and grouped, leaving a remainder.



Extend to numbers with 4 digits.

Interpret non-integer answers to division by expressing results in different ways according to the context

Bus stop with decimal remainders

$$\begin{array}{r} 112. \\ 5 \overline{)564.0} \\ 564 \div 5 = \end{array}$$

$$\begin{array}{r} 112. \\ 5 \overline{)564.0} \\ 564 \div 5 = \end{array}$$

Here a decimal point is placed next to the units and a zero placed in the tenths column. The remainder is carried over to make 40. 40 is then divided by 5 to give 8. The remainder is .8. This would be appropriate if solving a division that involved money or measures. For example, I share £564 between my 5 friends. How much will each friend get? The answer including the decimal would be £112.80

Bus stop with a fraction remainder

$$\begin{array}{r} 112.8 \\ 5 \overline{)564.0} \\ 564 \div 5 = 112.8 \end{array}$$

$$\begin{array}{r} 112 \\ 5 \overline{)564} \text{ r } 4/5 \\ 564 \div 5 = 112 \text{ r } 4/5 \end{array}$$

The previous example gave the remainder of 4. The fraction remainder would be $4/5$

Use multiplication and division as inverses

Solve problems involving division including scaling down

Divide whole numbers and those involving decimals by 10, 100 and 1000

Fluency-mental maths targets for division

- Count backwards in steps of powers of 10 for any given number up to 1,000,000
- Count backwards with positive/negative whole numbers through zero
- Practise mental calculation with increasingly large numbers
- Apply all multiplication tables and related division facts frequently, commit them to memory and use them to confidently make larger calculations

Year 6

Divide numbers by up to 4-digits by a 2-digit whole number using formal written methods of long division, interpret remainders as whole numbers, fractions or by rounding, as appropriate for the context

$$\begin{array}{r} 2 \\ 24 \overline{)560} \\ - 48 \\ \hline 8 \end{array}$$

Step 1-Work out how many 24s are in 56. There are 2, this is written above the bus stop and the 48 is written below. The 48 is subtracted from 56 to find the remainder.

$$\begin{array}{r} 2 \\ 24 \overline{)560} \\ - 48 \downarrow \\ \hline 80 \end{array}$$

Step 2-The zero is brought down next to the 8 to make 80.

$$\begin{array}{r} 23 \\ 24 \overline{)560} \\ - 48 \downarrow \\ \hline 80 \\ 72 \\ \hline 8 \end{array}$$

Step 3-Work out how many 24s there are in 80. There are 3 which is written above the units. The 72 is subtracted from 80 to work out the remainder.

$$\begin{array}{r} 23 \text{ r}8 \\ 24 \overline{)560} \\ - 48 \downarrow \\ \hline 80 \\ - 72 \\ \hline 8 \end{array}$$

Step 4-Interpret the remainder for the context. Here the remainder is 8.

$$\begin{array}{r}
 23 \frac{8}{24} \text{ (1/3)} \\
 24 \overline{)560} \\
 \underline{-48} \\
 80 \\
 \underline{-72} \\
 8
 \end{array}$$

8 out of 24, which gives us the fraction remainder. This can be cancelled down into 1/3.

Divide numbers with up to 2 decimal places by 1-digit and 2-digit whole numbers, initially in practical contexts involving money and measures

Understand the relationship between unit fractions and division

Recognise division calculations as the inverse of multiplication

Solve problems involving division

Fluency-mental maths targets for division

- Practise division for larger numbers, using the formal written methods of short and long division
- Continue to use all multiplication tables and division facts to maintain fluency
- Perform mental calculations, including with mixed operations and larger number